What does mathematics have to do with common sense? However disconnected these may seem to students entering my courses, I endeavor to persuade them thoroughly to the contrary. To understand a mathematical idea is to internalize the logical steps leading to it from our common sense. When this is accomplished, mathematics not only becomes easier and clearer; it becomes *more fun*. The joys of discovery and deep understanding appear to be universal. Of everything we consider common sense, we tend to appreciate most those things we can remember *not* understanding, especially if we also remember when they first became clear and how that felt.

Opportunities for that feeling, and the new knowledge that sparks it, are a central goal of my teaching. Students cannot achieve it, however, without failing many times along the way. When students are stuck, I seek to guide them not just toward correct answers, but toward a growth mindset. I remind students that being stuck is not only acceptable, but an essential part of the learning process. To communicate this point, one activity I use is to ask each student to answer the two questions “What is one activity you are very good at?” and “How did you get good at that activity?” The second question, in particular, is likely to yield a common theme of *practice*, which I hope will steer many students away from a fixed mindset (“I’m just not a math person” or “I can’t figure this out”) toward a growth mindset (“I need more practice” or “I *haven’t* figured this out *yet*”). I have never thought any concept in a course I’ve taught was out of any student’s reach, and hearing this can be encouraging to students who may feel lost or confused.

I find I am best able to promote a growth mindset when I can observe students actively attempting a problem, both in class and during office hours. In fact, my favorite office hour sessions are those in which students come in feeling the *most* stuck. This affords me the greatest opportunity to understand their thought process, built all the way up from common sense to a new discovery or a deepened understanding. I aspire to the same in the classroom, often utilizing the think-pair-share model, which benefits students’ communication skills as well as their mathematical understanding. A favorite example of this approach is when I introduce concavity. I assign each small group a basic qualitative graph shape, asking them to determine whether each of the first two derivatives is positive, negative, or zero. Students invoke what they know, from the course as well as their common sense, tracing the logic toward their conclusion, then comparing their answers with a nearby classmate. After discussing each conclusion together as a class until it makes sense to the students, I invite them to look for a pattern among the shapes based on their second derivative. The descriptions students offer are generally on the right track, and only then do I introduce the terms “concave up” and “concave down.”

I continue to expand the range of tools I use to facilitate interaction, a process which has largely been *helped* by the pandemic. Since it began, I’ve built up a supply of short, digestible lecture videos which can be found on my website. This makes the content seem less overwhelming than it would be in larger pieces, and affords students the convenience of being able to pause, rewind, and alter the speed of videos. I now solicit feedback specific to each video, so as to improve upon those that could benefit most from being reworked. Perhaps most importantly, this frees up more class time for meaningful interaction. In the current environment, since paper and pencil are difficult to see through students’ Zoom cameras, I have asked those students who do *not* possess electronic writing tools to get themselves a whiteboard with markers and erasers. These tools, both the physical and the electronic, allow students to smoothly share their written thoughts with classmates and with me, prompting an immediate back-and-forth which bypasses much of the second-guessing students may do when working alone.

A long-standing catalyst of this second-guessing, which I am actively working to remedy, is the single-attempt nature of traditional grading. Little by little, I am moving more and more of the structure of each course toward Mastery-Based Grading, a system in which students get several attempts to demonstrate mastery of each course objective and only their most successful attempt counts in their final grade. Having multiple attempts reduces students' anxiety, encouraging them to learn and grow from their mistakes and ensuring that when they do, it will reflect in their grade. This approach requires me to make up a lot more problems than ever before, but as each problem is graded on a shorter scale, I do a lot less hair-splitting over partial credit. My application of Mastery-Based Grading to quizzes has been well received thus far, and I am exploring ways to apply it to exams as well.

While striving to understand students' thought processes, and aid them in understanding their own, I have experimented with various strategies to meet each student where they are. I like to imagine a math course as a stepladder for students to climb. While I cannot force them to ascend, I can make the process more fulfilling with the kind of ladder I build. In particular, I aim to make sure the bottom rung is within every student's reach. To help students gauge their understanding of prerequisite material, I often use either a sample final exam from the previous course or a set of diagnostic exercises. Once each student knows where the gaps are in their prerequisite preparation, these can often be addressed through additional videos. As you will see on my website, even though Calculus 1 is the lowest-level math course offered by Emory, I have made several videos addressing what I find are common misconceptions regarding prerequisite material. I have also found ways to artfully balance early class days of Calculus 1 between old and new content, often systematically grouping them by topic. For instance, although exponents are considered review material while logarithms are considered new, combining these into one class period not only saves time, but emphasizes the intimate connection between them.

Exponents and logarithms offer just one example of a common theme throughout mathematics: *parts that look very different often fit together in the same arrangement*. To help students recognize this shared "skeleton," I come up with visual displays that emphasize the *role* a number or expression plays in the structure being explored. Often, this involves drawing rectangular boxes, which could contain any number, expression, or function they like, while the structure relating the boxes to each other remains the same. In some cases, I assign a color to each role in a formula as I show repeated examples of its use; this accentuates the structural symmetry between problems that otherwise might not have seemed related. Some of my favorite examples of this can be found in the "Best Videos" section of my website, on topics ranging from the Chain Rule to Integration by Parts to "zooming in" on a curve (until it looks like a straight line) to illustrate the Definition of the Derivative. At a more advanced level, there are also several videos of me illustrating triple integrals by slicing a potato!

In conjunction with visual displays, I make good use of analogies, especially to ideas familiar to students in everyday life. For example, I often describe the core ideas of calculus in terms of the relationship between a car's odometer and its speedometer. Among the many applications of calculus, I mention this one early and often because it seems relatable to students. Similarly, when I introduce derivatives as described above, I point out that the flat appearance of the ground around us is due to how far zoomed in we are to Earth's surface. A minute or two later, when I show the results of zooming in on the origin of the absolute value function, I extend the analogy by asking "What if the Earth were a cube, and you lived at one of the edges or corners?" I have found ways to make Curl and Divergence more intuitive by imagining wind acting on a ping-pong ball or a clump of dust, and have done the same with the Intermediate Value Theorem and the naming choices one can make for unknown quantities. The former is nicely illustrated every time one has to cross the street, while the latter relates directly to other naming choices in everyday life. I always point out that, while naming decisions offer a lot of flexibility ("What's in a name?"), one must carefully avoid giving two different things the same name. To make this concern relatable, I need only mention a name that could refer to two or more on-campus locations (such as "Woodruff" in the case of Emory).

All my visual displays and analogies are designed to support one overarching goal: to draw the most direct path from students' common sense to a given concept. Any such path to a new idea expands a student's knowledge horizon, and with enough practice their common sense horizon will follow. Both of these will serve to undermine any anxiety a student may be harboring, giving way to the innate curiosity hidden underneath. This, in turn, will expand the student's curiosity horizon, turning unknown unknowns into known unknowns as the student asks new questions. As all three horizons expand, the feeling this delivers will stick with students long after the course ends, motivating them to continue seeking it out and sharing it as widely as possible.