The Oddball's Oddball: The Unusual life of Paul Erdős

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Figure: Paul Erdős as I remember him, Uncle Paul or Paul nagybasci.

Erdős Facts

- Over 1600 research papers.
- Over 189,000 citations.
- Over 500 different co-authors.
- For most of his later years he had no permanent home.
- His influence changed the nature of mathematical research from a sole endeavor to a social function where mathematicians openly discussed ideas with one another.

Books on his life.

1. The Man Who Loved Only Numbers by Paul Hoffman.

2. My Brain Is Open by Bruce Schecter.

3. The Boy Who Loved Math: The Improbable Life of Paul Erdős by Deborah Heilgman (author) and LeUyen Pham (illustrator).

Areas of Research Influence

- **Number Theory**
- **Geometry**
- **Combinatorics**
- **Graph Theory**
- **Probability**

His work has now had influence in other areas such as **computer science** and **topology**.

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At 75 he added C.D. (**counts dead!**)

Figure: Paul loved ping pong.

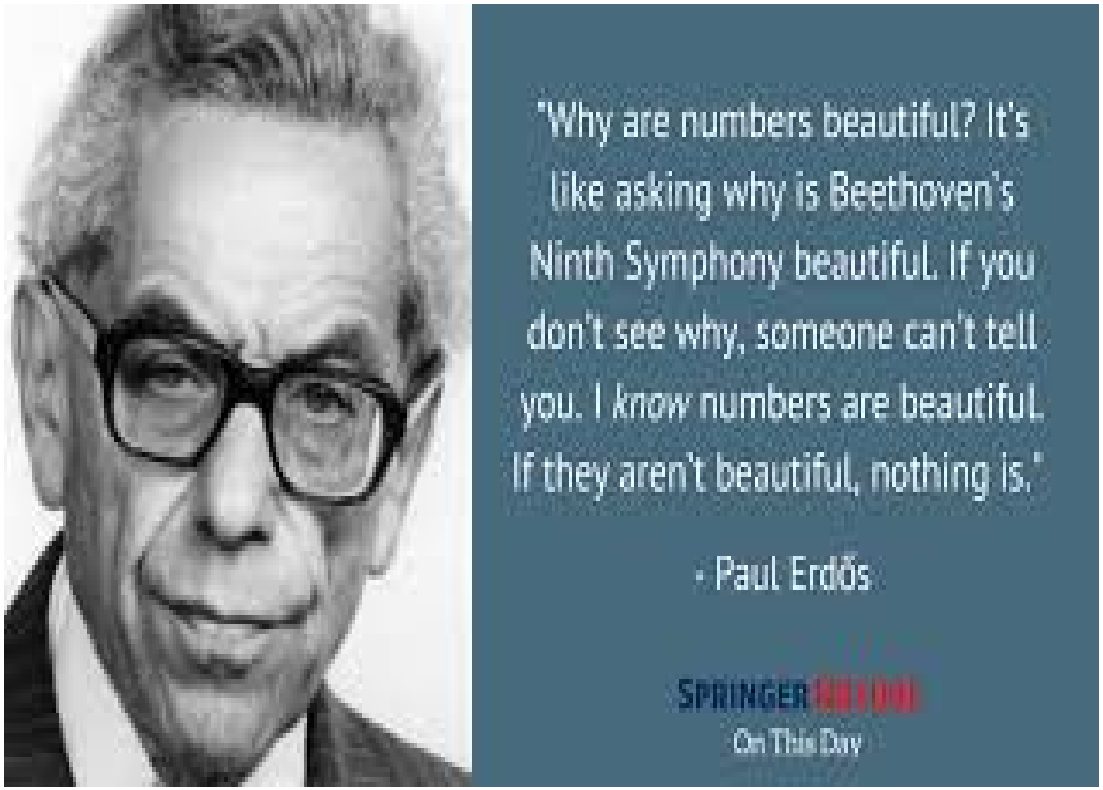
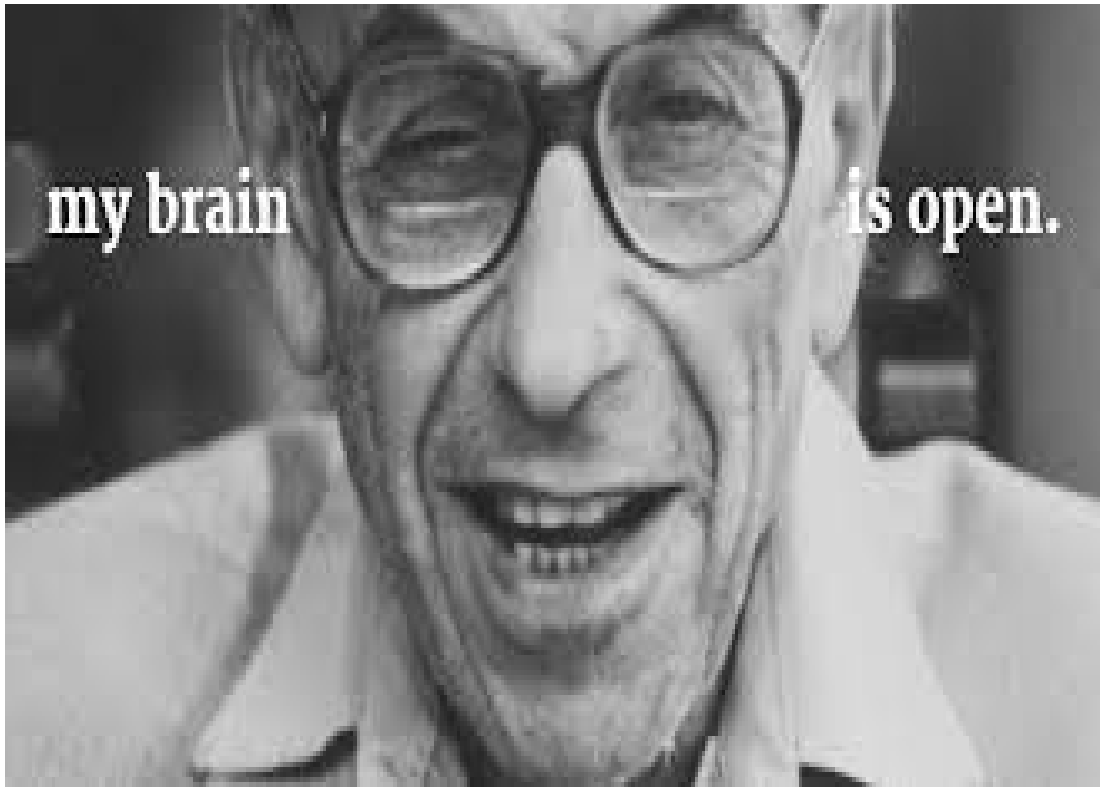


Figure: Paul quote.



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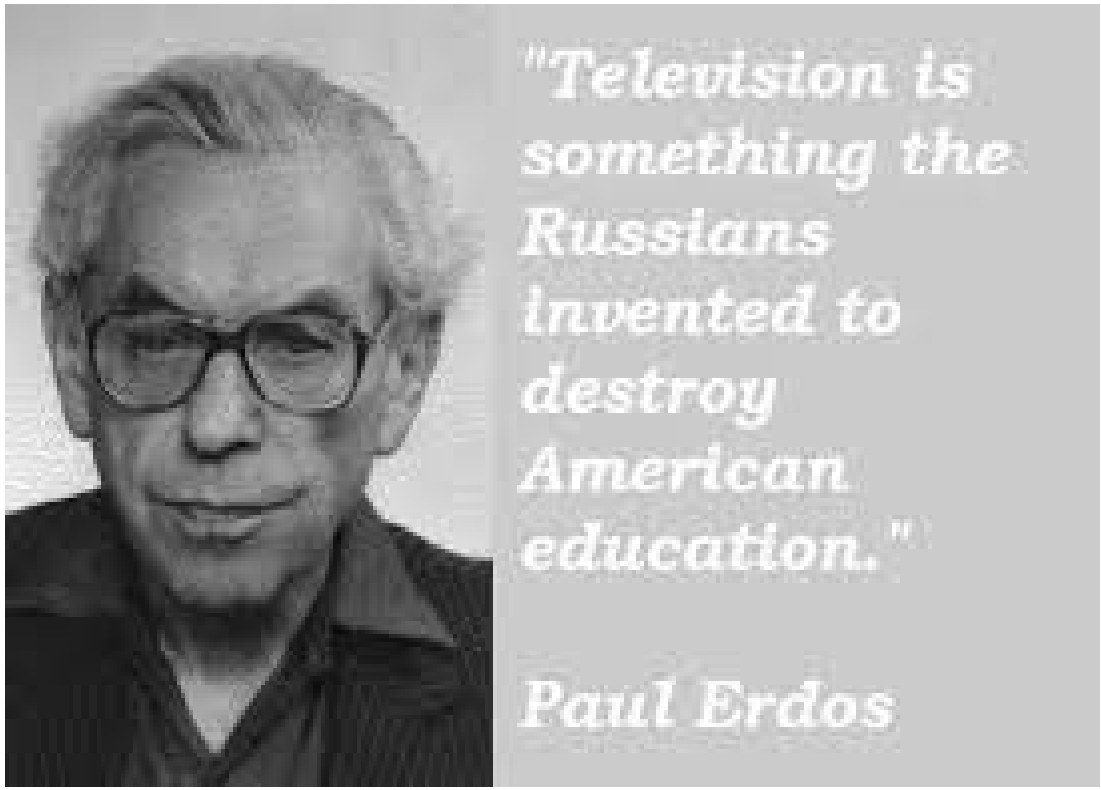

Figure: Paul quote.

Paul Quote - A little less famous

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Vicious hellhound please don't eat me, I am old and my meat is bad.

Paul Erdős in 1958.

Figure: Paul at 45.

Erdős Speak

Paul often joked that he wanted his epitaph to read:

I've finally stopped getting dumber.

More Erdős Speak

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The Book

The Book

Paul said the **SF** had a book which contained the best proofs of every Theorem.

It was his belief that mathematics is a fixed truth and we are only uncovering what must be. Sometimes the SF would let us get a peek into the book.

If he said you had the book proof for something, then you had the highest praise possible. Paul hoped the SF would let him read the book after he died, but expected to only get a glimpse or two.

Six degrees of Uncle Paul

Your **Erdős Number** is 1 if you wrote a paper with Paul.

It is 2 if you only wrote a paper with someone with Erdős Number 1, etc.

The **Erdős Number Project** is a website devoted to this with tons of info provided by the American Math Society. https://oakland.edua/enp/compute

The 90 Dancing Saints of St. Gregory of Nyssa Episcopal Church, San Francisco

This picture sits between that of Ghandi and Martin Luther.

Keep in mind that Paul did not dance!

Figure: Paul at St. Gregory.

More Erdős Facts

- He ran into trouble with SAM and JOE at various times.
- He believed strongly in personal liberty and fought all attempts to control his life in any way.

Understanding His Life - In the beginning

- Paul was born on March 26, 1913 in Budapest.
- He was the son of two high school math teachers.
- His parents were non-practicing Jews.
- His two older sisters died around the time of his birth from septic scarlet fever.

The Fear of Death

- His mother lived in constant fear Paul would contract something and die as well.
- He viewed Jews being beaten in the streets of Budapest when he was 6.
- Even as a teen he called himself old and declining and often spoke of death. He also greatly feared losing his mental abilities, even as a teen.

The child prodigy with a difficult life.

- His father was a Russian prisoner of war from 1914 to 1920. His mother was forced to work so he was left home alone a great deal.
- At age 3 he could multiply two 3-digit numbers in his head.
- At age 4 he discovered negative numbers on his own. He could also ask a person their birthdate and then compute how many seconds they had been alive.

- Taught himself to read by reading math text books.
- His mother often kept him home from school for entire years as she feared he would catch something fatal at school.

Figure: Paul as a child.

Figure: Paul with his mother Anna.

Start of a lifetime passion for prime numbers.

- His father taught him about prime numbers.
- A positive integer is **prime if the only divisors of the number are 1 and itself**.

  For example 2, 3, 5, 7, 11, 13 are all prime.
- Primes had long been a subject of interest to many mathematicians.

Legrendre (around 1800) first to predict a formula (no proof) on how the prime numbers were distributed. Refined by Gauss (1849).

The Prime Number Theorem.

Roughly, the number of primes less than or equal to the number N is about

$$\mathbf{N/ln\ N}.$$

This says that the probability an integer at most N is prime is about

$$\mathbf{1/ln\ N}$$

In 1949 Paul gave the first "elementary" proof of the Prime Number Theorem originally proved by **Hadamard** and **Poussin** in 1898.

The only proofs up to 1949 used very powerful deep mathematical results. Paul used only elementary facts.

Awards

1951 The American Math Society **Cole Prize in Number Theory**

1958 **Kossuth Prize** (Hungarian Science Award)

1983 **Wolf Prize**, at the time the most prestigious prize in mathematics with a $50,000 cash award.

At least fifteen honorary degrees including Emory in 1995.

Despite calling children epsilons, Paul loved children and was always willing to speak with them and entertain them.

Figure: Paul with Terry Tao, now a superstar mathematician at UCLA.

Some facts from his teen years

- Made a few friends as a teen, all of whom were strong math students. These included

Paul Turan
George Szerekes
Ester Klein
Tibor Gallai
Albert Renyi

All these people went on to have major research careers.

Figure: Paul with George Szekeres and Ester Klein, longtime friends.

Figure: Paul Turan

Opening an area of study

At 19 Paul meets Simon Sidon, a reclusive mathematician. Paul later describes Sidon as

"**a good mathematician, but a bit crazier than the average**."

Paul and Turan visit Sidon who gives them a problem:

What is the longest sequence $a_1, a_2, \ldots$ of positive integers less than or equal to N such that all the sums $a_i + a_j$ are distinct?

Paul says he does not know, but will return in a day or two with an answer. The answer took 20 years and is approximately $\sqrt{N}$.

Paul was a quick study!

Entered college at 17.

Graduated with a Ph.D. at 21.

A Brief Excursion into math - What is a graph???????

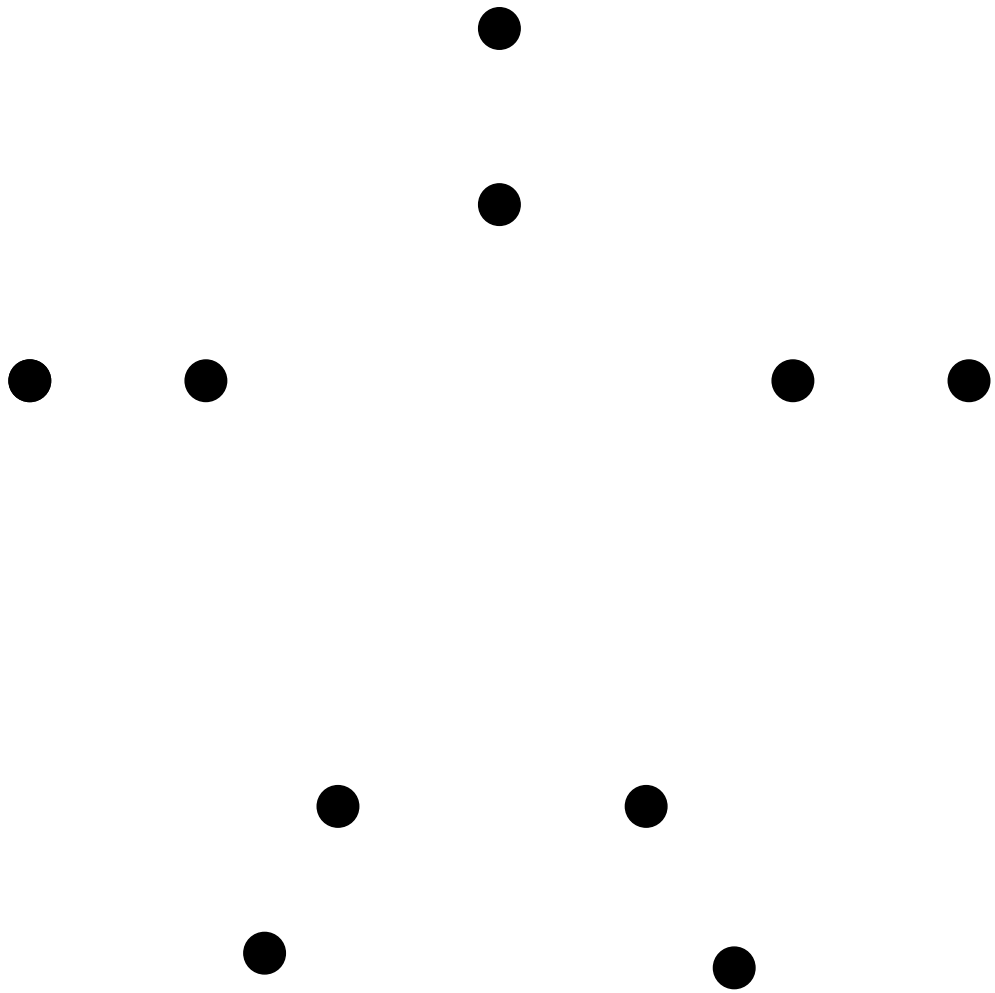

Figure: An example graph

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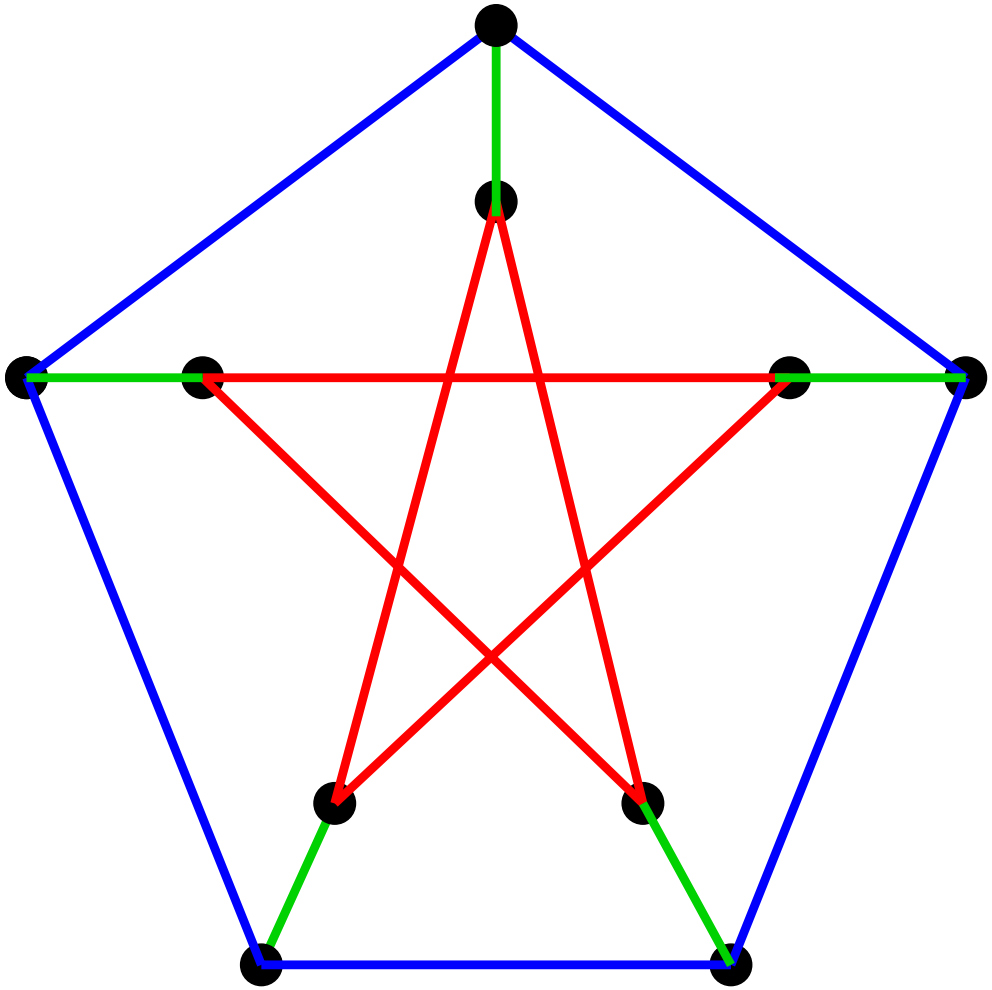


Figure: An example graph

What is Ramsey Theory

Frank Ramsey (1930) proved a remarkable result that essentially said that in any system of objects that is large enough, any given specific subsystem must exist.

How large the system must be depended on which subsystem you wanted.

Ramsey's result essentially says:

complete chaos cannot exist.

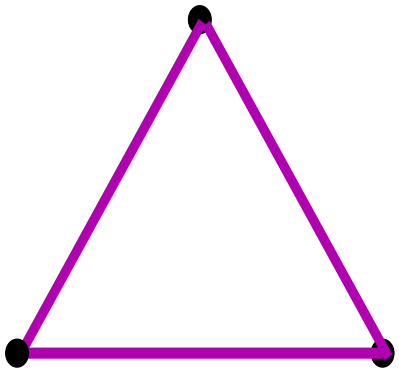
Erdős applied this principle to graphs many times, in the subarea called

Graph Ramsey Theory.

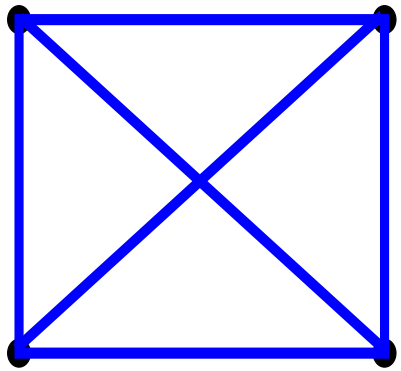
Graph Ramsey Theory

The first kind of graph people were interested in were the **Complete Graphs** with p vertices and all possible edges.

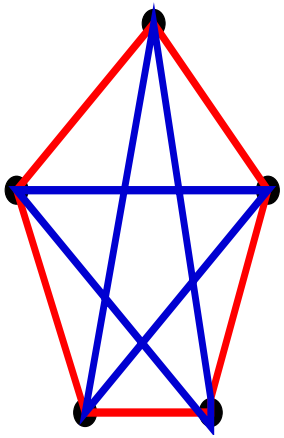
Example complete graphs



K_3



K_4



K_5

The most basic graph ramsey question is then:

How large a complete graph K_n is needed to ensure that no matter how we randomly color the edges of K_n with 2 colors, say red and blue, we either find a red triangle or a blue triangle?

This value n is denoted by $r(3,3)$.

It turns out that the difficulty in determining the exact ramsey number for complete graphs grows at a remarkable rate.

For triangles (K_3) it is: $r(3,3) = 6$

For K_4 it is: $r(4,4) = 18$

For K_5 it is unknown exactly. The best that is known is $43 \leq r(5,5) \leq 48$.

To find the exact value of $r(6,6)$ seems hopeless now.

Another major advance - The Probabilistic Method

Paul produced a remarkable result providing a lower bound on the ramsey number $r(k, k)$. He did this using a probabilty argument. That result still stands as the best lower bound known.

Roughly, Paul showed that $\mathbf{r(k, k) > 2^{k/2}}$.

Erdős - Renyi Random Graphs, 1959

There are two equivalent models:
Model 1: Consider each graph on N vertices as being equally likely to happen.

Model 2: With a fixed probabiity p insert an edge between vertices u and v and do this operation for all such pairs of vertices. The graph produced is called the random graph.

The start of a homeless life of constant travel

In 1938, sensing the coming war in Europe and the anti-semitic wave growing in Germany, Paul left Hungary for the United States.

He spent the next ten years in the U.S., starting with a fellowship at the Institute for Advanced Study at Princeton. There he met and often spoke with Einstein among many other great minds.

He also started traveling to other schools to do research with other people.

The start of trouble with SAM!!!

August 15, 1941: While attending a conference on Long Island, **Paul, Arthur Stone** (British), and **Shizuo Kakutani** (Japanese), decided to take a walk near the water during one of the breaks.

Drove to shore, stopped, started walking, deeply into their mathematics discussion and not noticing a keep out sign.

As they approached a short-wave radio station, a guard told them to leave. They immediately returned to their car and drove away.

Guard then panics and calls police saying 3 **Japanese spys** were trying to get to the station.

A short time later police arrested them while they were at lunch.

Newspaper headlines

Daily News: 3 Aliens nabbed at short-wave station

Post: FBI Defeats Spy Scare: 3 Aliens just students

They were questioned, released, and they returned to the conference.

But Paul now had an **FBI file!!!!!**

From the Institute he went on to teach at **Notre Dame** for three years. When asked if it bothered him to teach at a Catholic school Paul replied

No, but I find it strange how many plus signs they had displayed.

After 3 years at Notre Dame, he went to **Madison**, to teach at Wisconson, and to work with **Stan Ulam**.

1948 visit home - Good News - Bad News

Returning home he saw that his family had suffered greatly during the war. His father died of a heart attack in 1942. Four uncles had also been killed by the Germans.

Some good news

His mother, Aunt Freda and best friend Paul Turan were alive.
Turan and Aunt Freda had both survived the labor camps.

More Good News

He soon meets Turan's future wife, Vera Sós, also a world class mathematician.

While in the labor camp, Turan proved a major result that stimulated a deep and important subarea of graph theory, today called **Turan type extremal theory**.

The start of trouble with JOE

His stay in Hungary was short-lived. Stalin began a crackdown causing Paul to flee to England. He bounced back and forth between England and the U.S. (having managed to get back in) and he again accepted a position at Notre Dame in 1952.

Turned down a generous offer for a permanent position there, teaching one class per semester, and with a grad student assistant who would take over his teaching at the drop of a hat, should he feel the need to travel.

Troubles continue

In 1954 Paul was invited to the **International Mathematics Congress,** a very prestigous meeting held every 4 years. The meeting was in Amsterdam and Paul would need a re-entry permit to return to the U.S.

Questions by immigration included:

- Does your mother have great influence in the Hungarian government? (answer **NO**)
- Have you read Marx, Engels, or Stalin? (answer **NO**)

- **What do you think of Marx?**
- Answer: **I am not competent to judge, but no doubt he was a great man.**
- **Do you plan to visit Hungary?**
- Answer: **No, they might not let me back out. I am planning to visit England and Holland.**

Reentry permit **denied!!!** (What a surprise!)

Paul went anyways.

He then began bouncing between England and Holland and finally got a 3 month position at the **Hebrew University** in Jerusalem.

Became a resident of Israel, but not a citizen.

1955, got back into Hungary when well-connected friends argued that he was a singular asset to the world culture of mathematics.

He was granted a **special passport** that only he had - which allowed him to leave and reenter Hungary any time he wanted.

This was continued even after the Soviet crackdown in 1956.

By 1959 a letter writing campaign by mathematicians got Paul readmitted to the U.S.

Thus began a productive period where he would meet and collaborate with many new mathematicians in the U.S.

Paul's U.S. caretakers

In 1963 Paul met Ron Graham, who later became one of two primary caretakers for Paul in the U.S.

In 1973, Paul went to Memphis to work with Ralph Faudree, who became the other primary caretaker in the U.S.

Paul's Keepers in New Jersey

Figure: Paul with Ron Graham and Fan Chung.

2nd home in Memphis

Figure: Ralph Faudree

Figure: With Paul and the Memphis group, 1984.

Figure: Same group.

Figure: Introducing Paul to Hank Aaron, with Gary Hauk.

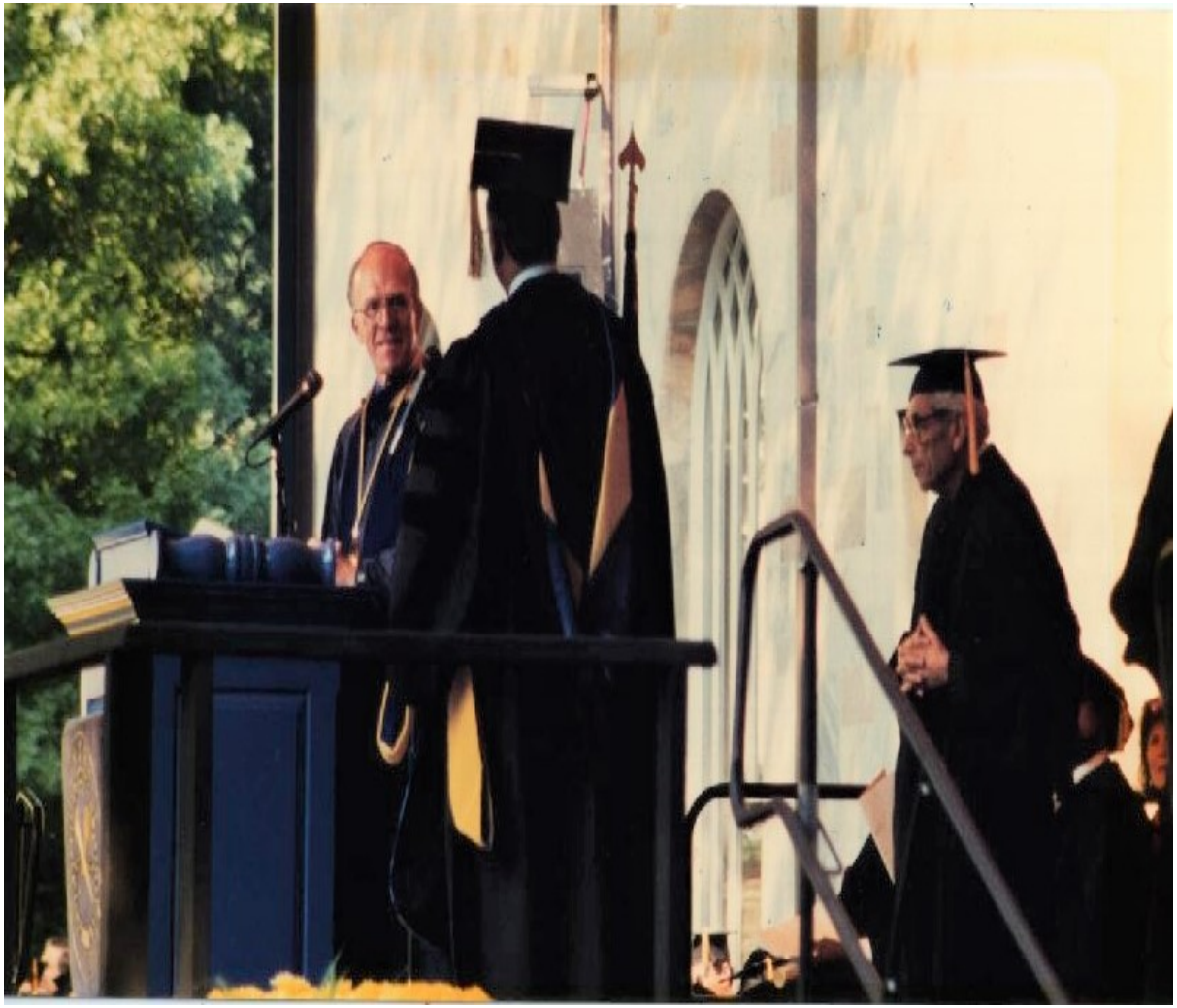


Figure: Paul receiving his honory degree from Emory, 1995.