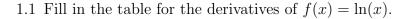
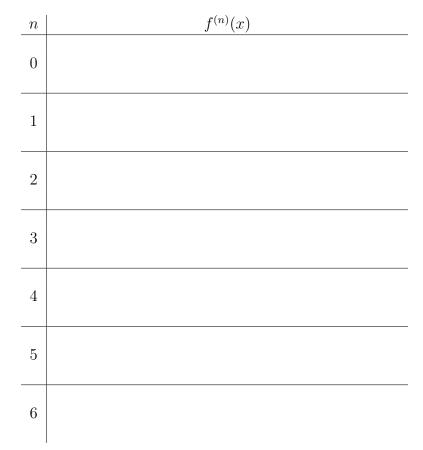
Groupwork 04/08: MATH 112 Prof. Maxwell Auerbach

Show all work. No credit will be given for answers without sufficient work. No calculators are allowed.

1 In Class Problems: Taylor Series Remainder Estimates

Recall the definition of a Taylor series centered at $a : \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k$





1.2 Let $f(x) = \ln(x)$. Use the definition of Taylor series to write the first four terms of the Taylor series for f(x) centered at a = 1. (This is called the fourth degree Taylor Polynomial)

Theorem: (Taylor Series Remainder Estimate pg. 762). Let $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)(a)}}{k!} (x-a)^k$ be a Taylor Series. Then for x in the interval [a-d, a+d], the following is true:

$$|R_n(x)| \le \max_{[a-d,a+d]} \frac{|f^{(n+1)}(x)|}{(n+1)!} |x-a|^{n+1}$$

where R_n is the remainder, or error, defined as

$$R_n(x) = \sum_{k=n+1}^{\infty} \frac{f^{(k)(a)}}{k!} (x-a)^k$$

for the Taylor polynomial

$$T_n(x) = \sum_{k=0}^n \frac{f^{(k)(a)}}{k!} (x-a)^k$$

An error estimate question will ask for the maximum error of the first n terms of a Taylor series in some range. We can think of the remainder estimate as finding the worst possible next term in that range. We can simplify this more. (n + 1)! does not change with x. The maximum of |x - a| over [a - d, a + d] is d. So we get the simplified Remainder estimate on the back page.

Theorem: (Taylor Series Remainder Estimate pg. 762). Let $f(x) = \sum_{k} = 0^{\infty} \frac{f^{(k)(a)}}{k!} (x-a)^{k}$ be a Taylor Series. Then for x in the interval [a-d, a+d], the following is true:

$$|R_n(x)| \le \frac{\max_{[a-d,a+d]} |f^{(n+1)}(x)|}{(n+1)!} |d|^{n+1}$$

To find the remainder estimate all we need to find is the maximum of the absolute value of the n + 1 derivative over [a - d, a + d]. Below we recap that process.

We want the maximum error of the fourth degree Talyor polynomial for $f(x) = \ln(x)$ over [9/10, 11/10].

- 1.3 What is d?
- 1.4 We need to find $\max_{[a-d,a+d]} |f^{(5)}(x)|$ (since n = 4). The maximum of a function over a closed interval is either at a critical point or the endpoints of the interval.
 - 1.4 a) To find the critical points of $f^5(x)$ we need to find where its derivative is zero or undefined and which, if any, are in the interval [a d, a + d]. Where is $f^6(x)$ zero or undefined? Which, if any, are in the interval [9/10, 11/10]
 - 1.4 b) Now find the values of $|f^{(5)}(x)|$ for x = 9/10, x = 11/10 and any critical points you find inside the interval [9/10, 11/10]. Which absolute value is the greatest?
- 1.5 Using your answer from 1.3 and 1.4 b), and using the formula

$$|R_n(x)| \le \frac{\max_{[a-d,a+d]} |f^{(n+1)}(x)|}{(n+1)!} |d|^{n+1}$$

find the maximum error of the fourth degree Talyor polynomial for $f(x) = \ln(x)$ over [9/10, 11/10].

1.6 Find the fourth degree polynomial for $f(x) = \cos(2x)$ about $a = \pi/3$, and the maximum error over $[0, 2\pi/3]$.

1.7 Find the fourth degree polynomial for $f(x) = xe^{(x)}$ about a = 1, and the maximum error over [0, 2].