

# Homework Week X

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Foundations of Mathematics

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**Problem 1.** Suppose that  $n$  is an even integer, and let  $m$  be any integer. Prove that  $nm$  is even.

*Proof.* Delete this and put your solution here. □

**Problem 2.** Suppose that  $n$  is an odd integer. Prove that  $n^2$  is an odd integer. (Hint: an integer  $n$  is odd if and only if there exists an integer  $k$  such that  $n = 2k + 1$ .)

*Proof.* Delete this and put your solution here. □

**Problem 3.** Prove that if  $n^2$  is even, then  $n$  is even. (Hint: see Section 1.4)

*Proof.* Delete this and put your solution here. □

**Problem 4.** Negate the following sentences.

1. All triangles are isosceles.
2. Every door in the building was locked.
3. Some even numbers are multiples of three.
4. Every real number is less than 100.
5. Every integer is positive or negative.
6. If  $f$  is a polynomial function, then  $f$  is continuous at 0.
7. If  $x^2 > 0$ , then  $x > 0$ .
8. There exists a  $y \in \mathbf{R}$  such that  $xy = 1$ .
9.  $(2 > 1)$  and  $(\forall x, x^2 > 0)$
10.  $\forall \epsilon > 0, \exists \delta > 0$  such that if  $|x| < \delta$ , then  $|f(x)| < \epsilon$ .

Here are a few LaTeX commands you might need. Delete this part before you submit your assignment

- $a \neq b$
- $n^2 + m$
- $\forall n \in \mathbb{Z}, \exists m \in \mathbb{N}$  such that ..