

MATH 250 HANDOUT 7 - MORE PROBLEMS ABOUT SETS; POWER SETS

- (1) Let A, B , and C be sets. Draw a Venn diagram demonstrating each of the following and then prove each of the following.
- (a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
 - (b) Prove that $A - (B \cap C) = (A - B) \cup (A - C)$.
 - (c) Prove that $(A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$.
 - (d) Prove that if $A - B \subseteq C$, then $\overline{C} \subseteq \overline{A} \cup B$.
 - (e) Prove that if $A \subseteq B$, then $A \cup B - A \cap B = A - B$.
- (2) Disprove each of the following statements by giving examples of sets A, B, C demonstrating that the statement is false. (I.e., give a counterexample.)
- (a) For all sets A, B, C , $A \cup (B \cap C) = (A \cup B) \cap C$.
 - (b) For all sets A, B, C , $A \cap (B \cup C) = (A \cap B) \cup C$.
 - (c) For all sets A, B, C , if A, B are subsets of C , then $(C - A) - B = C - (A - B)$.
 - (d) For all sets A, B, C , if A, B are subsets of C and if $A \subseteq B$, then $(C - A) \subseteq (C - B)$.

Power Sets

- (a) Write out 4 elements of
- (i) $P(\{0, 1, 2, 3, 4\})$;
 - (ii) $P(\mathbf{N})$;
 - (iii) $P(\mathbf{R})$.
- (b) Let $A = \{0, 1, 2\}$. Circle whichever of the following statements are true.
- (i) $\{0\} \subseteq P(A)$;
 - (ii) $\{1, 2\} \in P(A)$;
 - (iii) $\{\{0, 1\}, \{1\}\} \subseteq P(A)$;
 - (iv) $\emptyset \in P(A)$;
 - (v) $\emptyset \subseteq P(A)$;
 - (vi) $\{\emptyset\} \in P(A)$.
 - (vii) $\{\emptyset\} \subseteq P(A)$;
 - (viii) $\{1, \{1\}\} \subseteq P(A)$.
- (c) Let A and B be sets. Prove or disprove:
- (i) $P(A) \cup P(B) \subseteq P(A \cup B)$.
 - (ii) $P(A \cup B) \subseteq P(A) \cup P(B)$.
 - (iii) $P(A) \times P(B) \subseteq P(A \times B)$.
 - (iv) $A = B$ if and only if $P(A) = P(B)$.