

## MATH 250 HANDOUT 9 - FUNCTIONS AND IMAGES

(1) Give an example of a function with each of the following domain/codomains.

(a)  $\mathbf{Z} \rightarrow \mathbf{Z} \times \mathbf{Z}$ .

(b)  $\mathbf{Z} \times \mathbf{Z} \rightarrow \mathbf{Z}$ .

(c)  $\mathbf{Z} \rightarrow \mathbf{Z}_{>0}$ .

(d)  $\mathbf{R} \rightarrow \mathbf{Z}$ .

(e)  $P(\mathbf{R}) \rightarrow P(\mathbf{Z})$ .

(f)  $\mathbf{Z} \rightarrow P(\mathbf{Z})$ .

(g)  $P(\mathbf{Z}) \rightarrow \mathbf{Z}$ .

(2) Draw a picture of two different functions from  $\{1, 2, 3\} \rightarrow \{4, 5\}$ .

**Images**

- (3) Let  $f: A \rightarrow B$  be a function. Finish the following sentence: an element  $b \in B$  is not in the image of  $f$  if  $\dots$
- (4) Compute the image of the following functions:
- (a)  $g: \mathbf{Z} \rightarrow \mathbf{Z}$ , where  $g(x) = 2n + 1$ ;
  - (b)  $g: \mathbf{R} \rightarrow \mathbf{R}$ , where  $g(x) = 2x + 3$ ;
  - (c)  $f: \mathbf{R} \rightarrow \mathbf{R}$ , where  $f(x) = -x^2 + 1$ ;
  - (d)  $\cos: \mathbf{R} \rightarrow \mathbf{R}$ ;
  - (e)  $\tan^{-1}: \mathbf{R} \rightarrow \mathbf{R}$ ;
- (5) Let  $f: A \rightarrow B$  be a function and let  $X, Y \subset A$ . Prove or disprove each of the following:
- (a)  $X \subset Y \Rightarrow f(X) \subset f(Y)$ .
  - (b)  $X \subset Y \Leftarrow f(X) \subset f(Y)$ .
  - (c)  $f(X \cup Y) \subset f(X) \cup f(Y)$ .
  - (d)  $f(X \cup Y) \supset f(X) \cup f(Y)$ .
  - (e)  $f(X \cap Y) \subset f(X) \cap f(Y)$ .
  - (f)  $f(X \cap Y) \supset f(X) \cap f(Y)$ .
  - (g) (HW)  $f(X) - f(Y) \subset f(X - Y)$ .
  - (h) (HW)  $f(X) - f(Y) \supset f(X - Y)$ .