

## MATH 250 HANDOUT 1 - LOGIC

1. Which of these are **statements**? (I.e., for which of these sentences is ‘true or false’ meaningful?)

- (1) Today it is raining.
- (2) What is your name?
- (3) Every student in this class is a math major.
- (4)  $2 + 2 = 5$ .
- (5)  $x + 1 > 0$ .
- (6)  $x^2 + 1 > 0$ .
- (7) If it is raining, then I will wear my raincoat.
- (8) Give me that.
- (9) This sentence is false.
- (10) If  $x$  is a real number, then  $x^2 > 0$ .

2. Which of these are true?

- (1) (T or F) Every student in this class is a math major and a human being.
- (2) (T or F) Every student in this class is a math major or a human being.
- (3) (T or F)  $2 + 2 = 5$  or  $1 > 0$ .
- (4) (T or F) If  $x$  is a real number, then  $x^2 \geq 0$ .
- (5) (T or F) If  $x$  is a complex number, then  $x^2 \geq 0$ .

3. Write the negations of the following.

- (1)  $2 + 2 = 5$
- (2)  $1 > 0$ .
- (3)  $2 + 2 = 5$  or  $1 > 0$ .
- (4) Every student in this class is a math major.
- (5) Every student in this class is a math major or a human being.
- (6) If  $x$  is a real number, then  $x^2 > 0$ .

4. Prove the following using truth tables.

- (1)  $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$ ,
- (2)  $(P \vee Q) \vee R = P \vee (Q \vee R)$ . (We thus write  $P \vee Q \vee R$  for both.)
- (3)  $\neg(P \vee Q) = \neg P \wedge \neg Q$ ,
- (4)  $\neg(P \wedge Q) =$  (make a guess similar to problem 3),
- (5)  $\neg(\neg P) = P$ .

5. In exercise 6, you may use the following variants of exercise 4.

- (1)  $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$ ,
- (2)  $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$ . (We thus write  $P \wedge Q \wedge R$  for both.)
- (3)  $P \vee Q = Q \vee P$ .
- (4)  $P \wedge Q = Q \wedge P$ .

6. Prove or disprove the following *without* using truth tables.

- (1)  $\neg(P \wedge \neg Q) = \neg P \vee Q$ .
- (2)  $P \vee ((Q \wedge R) \wedge S) = (P \wedge Q) \vee (P \wedge R) \vee (P \wedge S)$ .
- (3)  $P \vee (Q \wedge R) \wedge S = (P \vee Q) \wedge (P \vee R) \wedge (P \vee S)$ .

7. Write the negations of the following implications.

- (1) If  $n$  is even, then  $n^2$  is even.
- (2) If  $1 = 0$ , then  $2 + 2 = 5$ .
- (3) If there is free beer, then DZB will drink it
- (4) If  $1 = 0$  and  $2 + 2 = 5$ , then the sky is blue and kittens are popular on youtube
- (5) If  $x$  and  $y$  are real numbers such that  $xy = 0$ , then  $x = 0$  or  $y = 0$ .

8. Which of these are true?

- (1) (T or F) For all  $x \in \mathbf{Z}$ ,  $x$  is divisible by 2.
- (2) (T or F) There exists an  $x \in \mathbf{Z}$  such that  $x$  is divisible by 2.
- (3) (T or F) For all  $x \in \mathbf{R}$ , if  $x \neq 0$ , then there exists a  $y \in \mathbf{R}$  such that  $xy = 1$ .
- (4) (T or F) For all  $x \in \mathbf{R}$ , there exists a  $y \in \mathbf{R}$  such that  $xy = 1$ .

9. Write the negations of the following.

- (1) For all  $x \in \mathbf{Z}$ ,  $x$  is divisible by 2.
- (2) There exists an  $x \in \mathbf{Z}$  such that  $x$  is divisible by 2.
- (3)  $\neg(\forall x, P(x))$ ,
- (4)  $\neg(\exists x \text{ s.t. } Q(x))$
- (5)  $\forall x, (P(x) \wedge Q(x))$ .
- (6) If  $\exists x \in \mathbf{R}$  such that  $2x = 1$ , then for all  $y$ ,  $y^2 < 0$ .
- (7) For all  $x \in \mathbf{R}$ , there exists a  $y \in \mathbf{R}$  such that  $xy = 1$ .

10. Write the converse and contrapositive of the statements from problem 7.