Comparing Reinforcement Learning to Optimal Control Methods on the Continuous Mountain Car Problem

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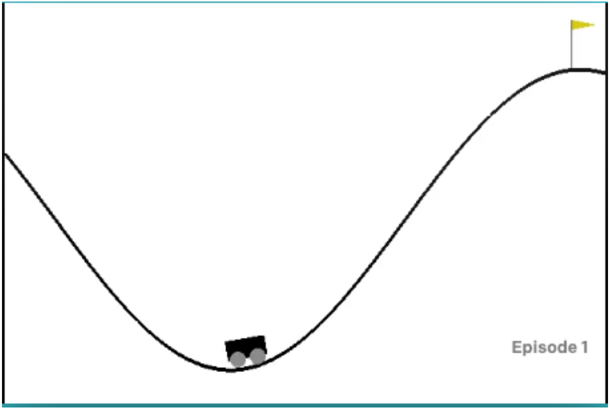
Continuous mountain car scenario



Continuous mountain car as optimal control problem

- The continuous mountain car can be modeled as

$$\partial_t \mathbf{z}(s) = \begin{pmatrix} z_2 \\ -\mu \cos(3z_1) \end{pmatrix} + \begin{pmatrix} 0 \\ \gamma \end{pmatrix} u(s), \quad \mathbf{z}(t) = z_t, \tag{1}$$

for $s \in (t, T]$ with the constants $\mu = 0.0025$, $\gamma = 0.0015$ and $\mathbf{z}(s) = \left(z_1, z_2\right)$.

- The unknown control $u : [t, T] \to [-1, 1]$ specifies the acceleration

- We seek to minimize the objective functional

$$J_{s,\mathbf{z}}[u] = \int_t^T L(s, \mathbf{z}, u)\, ds + g(\mathbf{z}(T)) \tag{2}$$

The running cost: $L(s, \mathbf{z}, u) = 0.01\|u(s)\|^2$
The terminal cost:

$$g(z) = \begin{cases} 0.45 - z_1, & z_1 < 0.45 \\ 0, & \text{otherwise.} \end{cases}$$

Why choose the mountain car?

- Established benchmark for RL models
- 2-D state-space allows for good plots and visualizations
- Both RL and optimal control problem
- Finite horizon (time)
- Continuous state and motion

Three Approaches:

1. Local solution using numerical ODE solvers and nonlinear optimization (baseline)
2. Reinforcement learning with actor-critic algorithm (data-based approach)
3. Optimal control using both model and data

Local Solution using Numerical ODE Solvers and Nonlinear Optimization

- To find the optimal control $\mathbf{u}_h$ we formulate an optimization problem
- We first discretize the control, state and the Lagrangian
- Setting $z_h^{(0)} = z_t$ and $\ell_h^{(0)} = 0$ allows us to use a forward Euler scheme for some control u

$$\begin{pmatrix} \mathbf{z}_h^{(i+1)} \\ \ell_h^{(i+1)} \end{pmatrix} = \begin{pmatrix} \mathbf{z}_h^{(i)} \\ \ell_h^{(i)} \end{pmatrix} + h \begin{pmatrix} f(t_i, \mathbf{z}_h^{(i)}) + \mathbf{B}\mathbf{u}_h^{(i)} \\ L(t_i, \mathbf{z}_h^{(i)}, \mathbf{u}_h^{(i)}) \end{pmatrix}, \quad i = 0, \ldots, N-1.$$

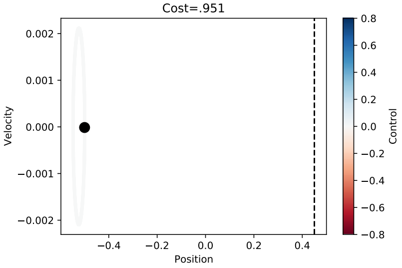
- Approximate our objective function as

$$J_{\mathbf{z}}[u] \approx J_{\mathbf{z}_h}(\mathbf{u}_h) = \ell_N + g(\mathbf{z}_h^{(N)})$$

- Yields the optimization problem

$$\min_{\mathbf{u}_h, \mathbf{z}_h} \ell_N + g(\mathbf{z}_h^{(N)})$$

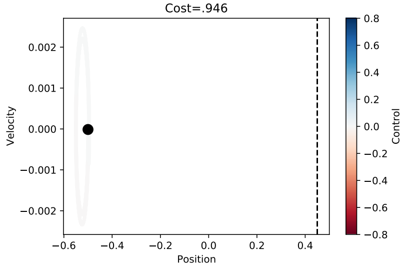
Gradient descent and corresponding trajectory plot



- To solve our optimization problem we use gradient descent. Take an initial guess for u_h and repeatedly update u_h using the gradient of the objective function and step size α

$$(u_h)_0 = \vec{0}$$

Gradient descent and corresponding trajectory plot



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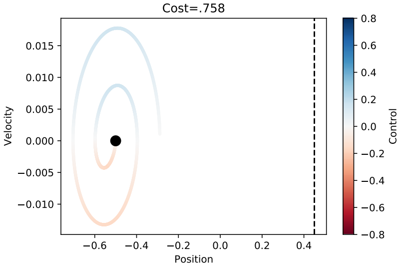
$$(u_h)_0 = \vec{0}$$
$$(u_h)_1 = (u_h)_0 - \alpha(\nabla J((u_h)_0))$$

Gradient descent and corresponding trajectory plot



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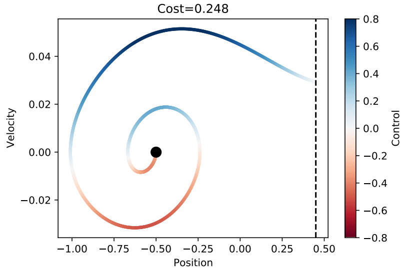
$$(u_h)_0 = \vec{0}$$

$$(u_h)_1 = (u_h)_0 - \alpha(\nabla J((u_h)_0))$$

$$\vdots$$

$$(u_h)_6 = (u_h)_5 - \alpha(\nabla J((u_h)_5))$$

Gradient descent and corresponding trajectory plot



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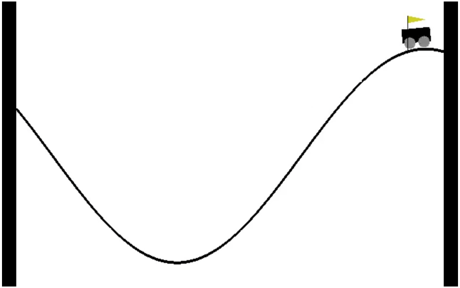
$$\vdots$$

$$(u_h)_6 = (u_h)_5 - \alpha(\nabla J((u_h)_5))$$

$$\vdots$$

$$(u_h)_* = (u_h)_{19} - \alpha(\nabla J((u_h)_{19}))$$

Optimal solution rendered (using local method)



Issues with the local method

- Dependent on starting location
- Unable to adapt to environment changes midway through
- Non-linear and non-convex problem which makes the method slower

Reinforcement learning

- Data-based approach to find global policy (for any initial condition)
- RL only considers the objective function and has no knowledge of the model
- Caveat: RL likes maximizing rewards so we will maximize negative cost
- Stochastic in two ways: initial position and action space
  - Allows for exploration
- We aim to estimate an optimal control policy $\psi : [t, T] \times \mathbb{R}^d \to \mathbb{R}^n$ via

$$\min_{\psi} \mathbb{E}_{z_t \sim \rho} \left(J_{\mathbf{z}}[u] \right) \quad \text{subject to} \quad (1).$$

TD-Advantage Actor-Critic

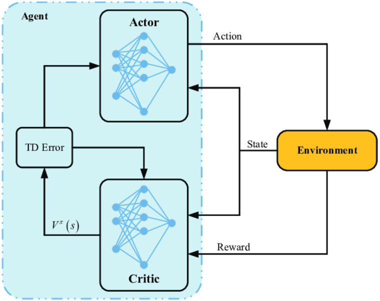


Figure: Actor-Critic model [1]

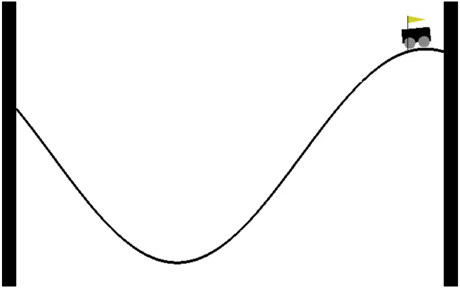
- Actor: $\psi^U(s, z_s) \approx \psi(s, z_s)$
- Critic: $V_\psi^V(z_s) \approx \mathbb{E}[J_{z_s}[\psi]]$
- TD Error:
  $\delta = r_{s+1} + V^\psi(z_{s+1}) - V^\psi(z_s)$
- Online algorithm
- Learns only from observations (data)

[1] H. Giang, T. Hoan, P. Thanh, and I. Koo. Hybrid noma/oma-based dynamic power allocation scheme using deep reinforcement learning in 5g networks. Applied Sciences, 10(12), 2020.

Modifications for RL and results

- Working in the OpenAI Gym mountain car environment
- Preexisting code
  - Changed rewards to be continuous
  - Fixed final time instead of ending iteration once reaching the goal
- Saw limited success from RL
- Rarely if ever found a solution
  - Most training cycles did not converge
- **Shortcoming of RL:** Fragile so we had hard time getting good solutions for OC problem

Suboptimal solution rendered (RL Method)



- Global solution is sub-optimal
- Took many episodes to get the car to the top of the mountain with our given conditions.

Optimal control method (preview)

- We aim estimate the value function

$$\Phi(t, z_t) = \min_u J_{t,\mathbf{z}}[u] \quad \text{subject to} \quad (1),$$

  which satisfies the Hamilton-Jacobi-Bellman PDE

- Our global optimal policy can be obtained through

$$\psi(s, \mathbf{z}) = \arg\max_u \mathcal{H}\left(s, \mathbf{z}, \nabla_z \Phi(s, z), u\right), \tag{3}$$

  where the Hamiltonian is defined as

$$\mathcal{H}(s, \mathbf{z}, p, u) = -p^\top \left(f(s, \mathbf{z}) + \mathbf{B}u\right) - L(s, \mathbf{z}, u).$$

- We will parameterize Φ using a neural network and train using the feedback form (3) and HJB

Summary

Progress:

- Adopted traditional continuous mountain car as control problem
- Found local solutions using numerical ODE solvers and nonlinear optimization
- Implemented actor-critic algorithm and found very suboptimal solutions, if a solution was even found

Next steps:

- Implement optimal control approach
- Optimize reinforcement learning method

Thank you! Any Questions?

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