Fast Training of Implicit Networks with Applications in Inverse Problems

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Introduction

Recent efforts in deep learning have turned towards solving inverse problems in imaging. For instance, Deep CNN was proposed for image denoising [12]. Moreover, the newly proposed implicit deep neural networks [2] are competitive with traditional feedforward networks on sequential data [1] and are effective in inverse problems in imaging [4]. Implicit networks backpropagate through a fixed point, which allows them to maintain constant memory costs. However, they are expensive to train since backpropagating through implicit networks requires the computation of a Jacobian-based linear system for every gradient evaluation. Recently, a Jacobian-Free Backpropagation (JFB) approach was proposed to avoid solving the Jacobian-based system [3], which adopts an approximation of the true gradient.

Implicit Deep Learning

Given a dataset $\{(d_i, x_i)\}_{i=1}^N \subset \mathbb{R}^n \times \mathbb{R}^n$, the relation between the ground truths x_i's and our measurements d_i's is represented by the forward model [8]:

$$d_i = \mathcal{A}x_i + \varepsilon \tag{1}$$

where $\mathcal{A}$ is a (non)linear measurement operator and ε is random **unknown** noise.
Our goal is to design a weight-tying neural network $\mathcal{N}_\Theta : \mathbb{R}^n \mapsto \mathbb{R}^n$ with K layers, where each layer $T_\Theta : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a (potentially nonlinear) mapping.
Given an input pair (d_i, x_i), we start with an initial guess x_i^0. Mimicking gradient descent and employing the forward model, we use the following updating rule [4]:

$$x_i^{k+1} = \underbrace{x_i^k - \eta\left(\nabla_x ||\mathcal{A}x_i^k - d_i||_{L^2}^2 + S_\Theta(x_i^k)\right)}_{:=T_\Theta(x_i^k)} \tag{2}$$

where $\eta > 0$ is the step size and $S_\Theta : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a trainable network that **learns** the gradient of an arbitrary regularizer. This is called the deep unrolling (DU) method.
For implicit networks, we expect the sequence $\{x_i^k\}_{k\in\mathbb{N}}$ to converge to a fix point x_i^* of T_Θ, i.e. $x_i^* = T_\Theta(x_i^*)$. This is true when T_Θ is a contraction mapping with Lipschitz constant $\gamma \in [0, 1)$.
Then we define

$$\mathcal{N}_\Theta(d_i) := x_i^* = T_\Theta(x_i^*) \tag{3}$$

as the output of our neural network, given an input d_i.
We can also choose other schemes to replace the iteration in Eq. 2, such as *proximal gradient descent* and *the alternating direction method of multipliers (ADMM)* [4].
Implicit neural networks can be trained using gradient descent and a calculated fix point. Suppose an experimenter chooses loss function ℓ. Then using implicit differentiation and Eq. 3 we have:

$$\frac{d\ell}{d\Theta} = \frac{d\ell}{d\mathcal{N}_\Theta}\frac{d\mathcal{N}_\Theta}{d\Theta} = \frac{d\ell}{d\mathcal{N}_\Theta}\frac{dx^*}{d\Theta} = \frac{d\ell}{d\mathcal{N}_\Theta}\left(I - \frac{dT_\Theta(x^*;d)}{dx^*}\right)^{-1}\frac{\partial T_\Theta(x^*;d)}{\partial\Theta} \tag{4}$$

Eq. 4 calculates the true gradient of our neural network parameters Θ with respect to loss function ℓ. However, calculating the inverse

$$\left(I - \frac{dT_\Theta(x^*)}{dx^*}\right)^{-1}$$

is **highly nontrivial** since a Jacobian-based linear system needs to be solved.

Jacobian-Free Backpropagation (JFB)

The goal of JFB is to **alleviate memory requirement** and **avoid high computational cost** in implicit networks. The key idea is to replace the problematic Jacobian $\left(I - \frac{dT_\Theta(x^*)}{dx^*}\right)$ in Eq. 4 with the identity matrix I. As a result, implicit networks are trained faster and more easily implemented—all while maintaining competitive results in image classification tasks [3].
We make the proposed substitution in Eq. 4 to approximate the gradient $\frac{d\ell}{d\Theta}$ and obtain:

$$p_\Theta = \frac{d\ell}{d\mathcal{N}_\Theta}\frac{\partial T_\Theta(x^*)}{\partial\Theta}$$

which is a descent direction for the loss ℓ.
Note: the JFB approach relies on more assumptions to hold:

- T_Θ is continuously differentiable w.r.t. Θ
- $M := \frac{\partial T_\Theta}{\partial\Theta}$ has full column rank.
- M is well-conditioned, i.e., $\kappa(M^TM) < \frac{1}{\gamma}$, where γ is the Lipschitz constant of T_Θ.

Results

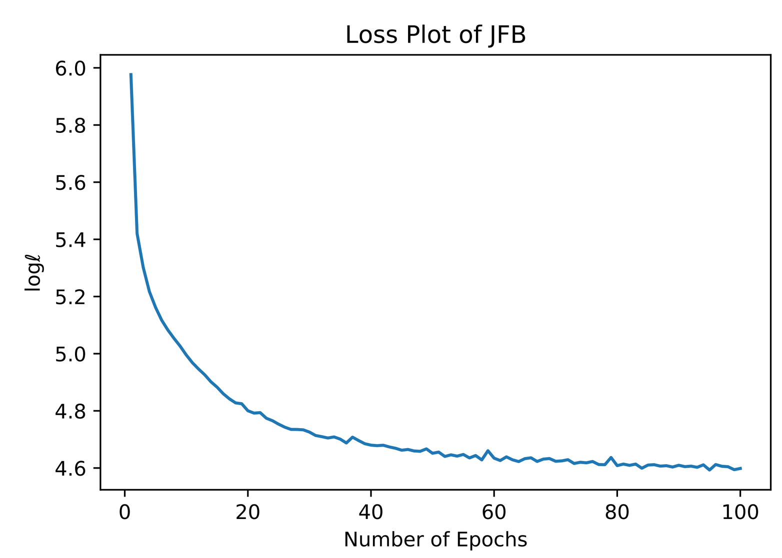

Fig. 1: Plot of Mean Squared Error (MSE) Per Image
step size $\eta = 10^{-3}$, learning rate $\alpha = 10^{-4}$

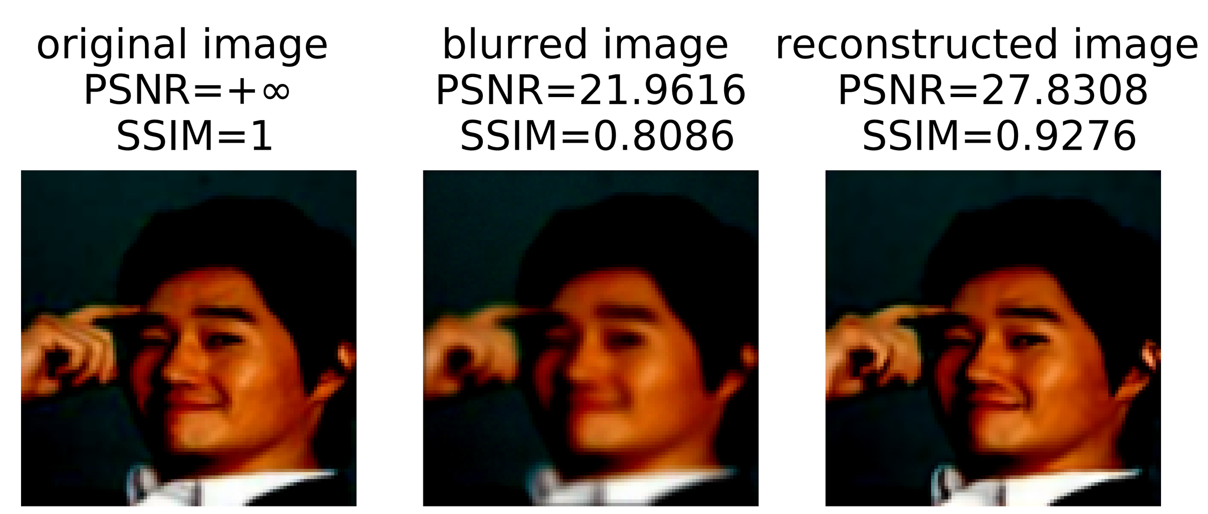

Fig. 2: Result of Proposed JFB on a Test Image Used by [4]

Note: Two metrics are commonly used for assessing the quality of reconstructed images [5]: the peak-signal-to-noise ratio (PSNR, a positive number, best at $+\infty$) and the structural similarity index measure (SSIM, also positive, best at 1).

Comparison

	Total Variation [9]	Plug-n-Play [10]	Deep Equilibrium [4]	JFB (Ours)
PSNR	26.79	29.77	32.43	27.83
SSIM	0.86	0.88	0.94	0.9276

The table above records the mean PSNR and SSIM of the dataset for our various models (statistics from [4]). It can be observed that applying JFB to training models for inverse problems in imaging is competitive.

Remarks

Our model is currently trained on a subset (8,000 images) of the CelebA dataset [7] using 1 NVIDIA RTX A6000 GPU.
Future directions include: (i) continuing to train current model until convergence
(ii) training JFB models on other schemes (proximal gradient descent & ADMM) as in [4] (iii) training JFB models on datasets such as fastMRI [6] [11]

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