Analysis and Differential Geometry Seminar

The dynamics of random soliton and soliton gasses

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Abstract: N. Zabusky coined the word "soliton" in 1965 to describe a curious feature he and M. Kruskal observed in their numerical simulations of the initial-value problem for a simple nonlinear PDE. The first part of the talk will be a broad introduction to the theory of solitons/solitary waves and integrable PDEs (the KdV and modified KdV equation in particular), describing classical results in the field. The second (and main) part of the talk will focus on some new developments and growing interest into a special case of solutions defined as "soliton gas".
I will describe a collection of works done in collaborations with K. McLaughlin (Tulane U.), T. Grava (SISSA/Bristol), R. Jenkins (UCF) and A. Minakov (U. Karlova). We analyze the case of a regular, dense (modified) KdV soliton gas and its large time behaviour with the presence of a single trial soliton travelling through it. We show that the solution can be decomposed as the sum of the background gas solution -a modulated elliptic wave-, plus a soliton solution: the individual expressions are however quite convoluted due to the nonlinear interaction.
We are able to derive a series of physical quantities that precisely describe the dynamics: the local phase shift of the gas after the passage of the soliton, the location of the soliton peak as the dynamics evolves, and the velocity of the soliton peak. In particular, we show that the peak velocity, while interacting with the gas, is highly oscillatory, while its leading-order, average value satisfies the kinetic velocity equation analogous to the one posited by V. Zakharov and G. El. I will finally present some ongoing work where we establish that the soliton gas is the universal limit for a large class of N-solutions with random initial data.

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