Combinatorics Seminar

Extremal problems for uniformly dense hypergraphs

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Abstract: Extremal combinatorics is a central research area in discrete mathematics. The field can be traced back to the work of Turán and it was established by Erd?s through his fundamental contributions and his uncounted guiding questions. Since then it has grown into an important discipline with strong ties to other mathematical areas such as theoretical computer science, number theory, and ergodic theory.
We focus on extremal problems for hypergraphs, which were introduced by Turán. After solving the analogous question for graphs, Turán asked to determine the maximum cardinality of a set E of 3-element subsets of a given n-element set V such that for any 4 elements of V at least one triple is missing in E. This innocent looking problem is still open and despite a great deal of effort over the last 80 years and our knowledge is still somewhat limited. We consider a variant of the problem by imposing additional restrictions on the distribution of the 3-element subsets in E. These additional assumptions yield a finer control over the corresponding extremal problem. In fact, this leads to many interesting and more manageable subproblems, some of which were already considered by Erd?s and Sós in the 1980ies. The additional assumptions on the distribution of the 3-element subsets are closely related to the theory of quasirandom discrete structures, which was pioneered by Szemerédi and became a central theme in the field. In fact, the hypergraph extensions by Gowers and by Rödl et al. of the regularity lemma provide essential tools for this line of research.

Monday, March 27, 2023, 4:30 pm
Mathematics and Science Center: MSC E408

Mathematics
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