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A volcanic approach to CM points on Shimura curves

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Abstract: A CM component of the ℓ-isogeny graph of elliptic curves has a particular structure, that of an ℓ-volcano, at least away from certain CM orders. The structure of "isogeny volcanoes" has seen much use in the study of CM elliptic curves over finite fields, originating with 1996 PhD thesis work of Kohel. Recent work of Clark—Saia leverages infinite depth versions of these graphs to study moduli of isogenies of CM elliptic curves over $\overline{\mathbb{Q}}$.
We will discuss an analogue of this work for abelian surfaces with quaternionic multiplication. A main result is an algorithm to compute the o-CM locus on the Shimura curve $X_0^D(N)$ over $\mathbb{Q}$, for o any imaginary quadratic order and $\gcd(D, N) = 1$. As an application, we give an explicit list of pairs (D, N) for which the Shimura curves $X_0^D(N)$ and $X_1^D(N)$ may fail to have a sporadic CM point.

Tuesday, March 21, 2023, 4:00 pm
Mathematics and Science Center: MSC W301

MATHEMATICS
EMORY UNIVERSITY