Dissertation Defense

Isogenies of Elliptic Curves and Arithmetical Structures on Graphs

Tomer Reiter
Emory University

Abstract: In this defense, we prove two results that come from studying curves. The first is a classification result for elliptic curves. Let $\mathbf{Q}(2^\infty)$ be the compositum of all quadratic extensions of $\mathbf{Q}$. Torsion subgroups of rational elliptic curves base changed to $\mathbf{Q}(2^\infty)$ were classified by Laska, Lorenz, and Fujita. Recently, Daniels, Lozano-Robledo, Najman, and Sutherland classified torsion subgroups of rational elliptic curves base changed to $\mathbf{Q}(3^\infty)$, the compositum of all cubic extensions of $\mathbf{Q}$. We classify cyclic isogenies of rational elliptic curves base changed to $\mathbf{Q}(2^\infty)$, for all but finitely many elliptic curves over $\mathbf{Q}(2^\infty)$.

Next, we turn to arithmetical structures, which Lorenzini introduced to model degenerations of curves. Let G be a connected undirected graph on n vertices with no loops but possibly multiedges. Given an arithmetical structure $(\mathbf{r}, \mathbf{d})$ on G, we describe a construction which associates to it a graph G' on $n-1$ vertices and an arithmetical structure $(\mathbf{r}', \mathbf{d}')$ on G'. By iterating this construction, we derive an upper bound for the number of arithmetical structures on G depending only on the number of vertices and edges of G. In the specific case of complete graphs, possibly with multiedges, we refine and compare our upper bounds to those arising from counting unit fraction representations.

Friday, March 19, 2021, 11:00 am
https://emory.zoom.us/j/92804829998?pwd=OFpZcWdlS2lrUFRLbDZQNklxZ3IwQT09

Advisor: David Zureick-Brown

Mathematics
Emory University