Algebra Seminar

Global mild solutions of the Landau and non-cutoff Boltzmann equation

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Abstract: This paper proves the existence of small-amplitude global-in-time unique mild solutions to both the Landau equation including the Coulomb potential and the Boltzmann equation without angular cutoff. Since the well-known works (Guo, 2002) and (Gressman-Strain-2011, AMUXY-2012) on the construction of classical solutions in smooth Sobolev spaces which in particular are regular in the spatial variables, has still remained an open problem to obtain global solutions in an $L^\infty_{x,v}$ framework, similar to that in (Guo-2010), for the Boltzmann equation with cutoff in general bounded domains. One main difficulty arises from the interaction between the transport operator and the velocity-diffusion-type collision operator in the non-cutoff Boltzmann and Landau equations; another major difficulty is the potential formation of singularities for solutions to the boundary value problem. In this work we introduce a new function space with low regularity in the spatial variable to treat the problem in cases when the spatial domain is either a torus, or a finite channel with boundary. For the latter case, either the inflow boundary condition or the specular reflection boundary condition is considered. An important property of the function space is that the $L^\infty_T L^2_v$ norm, in velocity and time, of the distribution function is in the Wiener algebra $A(\Omega)$ in the spatial variables. Besides the construction of global solutions in these function spaces, we additionally study the large-time behavior of solutions for both hard and soft potentials, and we further justify the property of propagation of regularity of solutions in the spatial variables. To the best of our knowledge these results may be the first ones to provide an elementary understanding of the existence theories for the Landau or non-cutoff Boltzmann equations in the situation where the spatial domain has a physical boundary.

This is a joint work with Renjun Duan (The Chinese University of Hong Kong), Shuangqian Liu (Jinan University) and Shota Sakamoto (Tohoku University).

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