## Algebra Defense

## Non-Archimedean and Tropical Techniques in Arithmetic Geometry

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**Abstract:** Let K be a number field, and let C/K be a curve of genus  $g \ge 2$ . In 1983, Faltings famously proved that the set C(K) of K-rational points is finite. Given this, several questions naturally arise:

- 1. How does this finite quantity #C(K) varies in families of curves?
- 2. What is the analogous result for degree d > 1 points on C?
- 3. What can be said about a higher dimensional variant of Faltings result?

In this thesis, we will prove several results related to the above questions.

In joint with J. Gunther, we prove, under a technical assumption, that for each positive integer d > 1, there exists a number  $B_d$  such that for each g > d, a positive proportion of odd hyperelliptic curves of genus g over  $\mathbb{Q}$  have at most  $B_d$  "unexpected" points of degree d. Furthermore, we may take  $B_2 = 24$  and  $B_3 = 114$ .

Our other results concern the strong Green–Griffiths–Lang–Vojta conjecture, which is the higher dimensional version of Faltings theorem (neé the Mordell conjecture). More precisely, we prove the strong non-Archimedean Green–Griffiths–Lang–Vojta conjecture for closed subvarieties of semi-abelian varieties and for projective surfaces admitting a dominant morphism to an elliptic curve.

Time permitting, we will introduce a new construction of the non-Archimedean Kobayashi pseudometric for a Berkovich analytic space X and provide evidence that our definition is the "correct" one. In particular, if this pseudo-metric is an actual metric on X, then it defines the Berkovich analytic topology and X does not admit a non-constant morphism from any analytic tori.

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## MATHEMATICS EMORY UNIVERSITY