Algebra and Number Theory Colloquium

The foundation of a matroid

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Abstract: Originally introduced independently by Hassler Whitney and Takeo Nakasawa, matroids are a combinatorial way of axiomatizing the notion of linear independence in vector spaces. If K is a field and n is a positive integer, any linear subspace of K^n gives rise to a matroid; such matroid are called **representable** over K. Given a matroid M, one can ask over which fields M is representable. More generally, one can ask about representability over partial fields in the sense of Semple and Whittle. Pendavingh and van Zwam introduced the **universal partial field** of a matroid M, which governs the representations of M over all partial fields. Unfortunately, most matroids (asymptotically 100%, in fact) are not representable over any partial field, and in this case, the universal partial field gives no information.

Oliver Lorscheid and I have introduced a generalization of the universal partial field which we call the **foundation** of a matroid. The foundation of M is a type of algebraic object which we call a **pasture**; pastures include both hyperfields and partial fields. Pastures form a natural class of field-like objects within Lorscheid's theory of ordered blueprints, and they have desirable categorical properties (e.g., existence of products and coproducts) that make them a natural context in which to study algebraic invariants of matroids. The foundation of a matroid M represents the functor taking a pasture F to the set of rescaling equivalence classes of F-representations of M; in particular, M is representable over a pasture F if and only if there is a homomorphism from the foundation of M to F. (In layman's terms, what we're trying to do is recast as much as possible of the theory of matroids and their representations in functorial "Grothendieck-style" algebraic geometry, with the goal of gaining new conceptual insights into various phenomena which were previously understood only through lengthy case-by-case analyses and ad hoc computations.)

As a particular application of this point of view, I will explain the classification which Lorscheid and I have recently obtained of all possible foundations for ternary matroids (matroids representable over the field of three elements). The proof of this classification theorem relies crucially on Tutte's celebrated Homotopy Theorem.

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