Equal sums of two cubes of quadratic forms: an apology

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Abstract: The topic of equal sums of two cubes has occupied number theorists and algebraists for a long time. In this talk, I will describe a one-parameter family of six binary quadratic forms $f_i$ so that $f_1^3 + f_2^3 = f_3^3 + f_4^3 = f_5^3 + f_6^3$ and so that every pair of equal sums of two cubes arises as one of the equalities here, perhaps with terms flipped. I will name-check Euler, Sylvester and Ramanujan. My favorite single example is

$$(x^2 + xy - y^2)^3 + (x^2 - xy - y^2)^3 = 2x^6 - 2y^6$$

The famous Euler-Binet parameterization of solutions over $\mathbb{Q}$ will be combined with point-addition of elliptic curve theory in what appears to be a novel way.

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