MATH 427: COMPLEX ANALYSIS (SUMMER 2018)

Note: In addition to Prof. Hart Smith's practice problems, consider the following problems.

(1) Evaluate the integral

$$\int_{|z-a|=a} \frac{z}{z^4 - 1} dz, \quad a > 1$$

(2) Evaluate the integral

$$\frac{1}{2\pi i} \int_{|z-a|=1} \frac{ze^z}{(z-a)^3} dz, \quad a \in \mathbb{C}$$

(3) Find the power series expansion of the following functions and determine the radius of convergence.

   (a) $f(z) = \dfrac{z^2}{(z+1)^2}$ at $z = 0$

   (b) $f(z) = \log \dfrac{1+z}{1-z}$ for any branch of $\log z$ at $z = 0$.

(4) Prove that if $f(z)$ is a noncostant function analytic on a connected open set U. Suppose f has no zeros in U, show that the minimum of $|f(z)|$ cannot be attained inside U.