MATH 427: COMPLEX ANALYSIS (SUMMER 2018)

Homework 3: due Monday, July 16th.

- Section 2.2: 11, 15.
- Section 2.3: 5, 9.
- Section 2.4: 9.

Additional problem:

(1) Use Cauchy-Riemann equations to determine whether the following functions are analytic on $\mathbb{C}$. In case the function f is analytic, find $f'(z)$:
  (a) z^2
  (b) $|z|^2$
  (c) $\cos(z)$

(2) Let $\gamma(t) = Re^{it}, 0 \leq t \leq 2\pi$ with R a positive constant. Compute the following integrals:

  (a) $\int_\gamma \bar{z} dz$

  (b) $\int_\gamma z^{-1} dz$

  (c) $\int_\gamma p(z) dz$ where $p(z)$ is any polynomial in z i.e.

$$p(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$$

  with $a_i \in \mathbb{C}, i = 1, \cdots, n$.

(3) If $f(z)$ is continuous on $\mathbb{C}$, show that

$$\lim_{R \to 0} \int_0^{2\pi} f(Re^{i\theta}) d\theta = 2\pi f(0).$$

Hint: consider proving $\lim_{R\to 0} \int_0^{2\pi} f(Re^{i\theta}) - f(0) dz = 0$. This problem implies that for γ as in #(2) above, we have

$$\lim_{R \to 0} \int_\gamma \frac{f(z)}{z} dz = 2\pi i f(0).$$