MATH250: FOUNDATIONS OF MATHEMATICS
SPRING 2020

Practice Problems Chapter 1: (please let me know if you find any typo.)

(1) Write the negation, converse, contrapositive of the following if possible:
  (a) For all $x \in \mathbb{R}$, there exists a $y \in \mathbb{R}$ such that $xy = 1$.
  (b) If x, y are real numbers such that $xy = 0$, then $x = 0$ or $y = 0$.
  (c) $P \wedge Q \implies R$ (Here, P, Q, R are statements.)

(2) The definition of a bounded function is the follows. A real valued function $f(x)$ is bounded on the interval $[a, b]$ if $f(x)$ is defined on $[a, b]$ and there exists a positive real number M such that $|f(x)| \leq M$ for all $x \in [a, b]$. What is the definition of unbounded function (functions that are not bounded).

(3) Write the truth table for $P \implies \neg Q$.

(4) Let $a, b, c \in \mathbb{Z}$. Show that if $ac|bc$ and $c \neq 0$, then $a|b$.

(5) Prove that $4 \nmid (n^2 + 2)$ for any integer n.

(6) Prove that if x and y are odd, then $x^2 + y^2$ is even but not divisible by 4.

(7) Prove that if $x + y > 5$, then $x > 2$ or $y > 3$.

(8) Prove that there are no positive integer solutions to the equation $x^2 - y^2 = 10$.

(9) Prove that there are no integer solutions of the equation
$$(x^2 - y^2)(x^2 - 4y^2) = 7.$$

(10) Prove that if $r^3 + r + 1 = 0$, then r is irrational.

(11) Let a, b, c, d be positive integers such that
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1$$
Prove that at least one of a, b, c, d is even.

(12) Show that if a is rational and b is irrational, then $a + b$ is irrational.

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(13) Prove that $\sqrt[3]{5}$ is irrational.

(14) Prove or disprove: let $a, b \in \mathbb{Z}, a, b \geq 1$. Then $\log_a b$ is irrational.

(15) Prove or disprove: let $p_1, p_2, \cdots, p_n$ be prime numbers. Then $p_1 p_2 \cdots p_n + 1$ is prime.

(16) Prove or disprove: there are no integer solutions to the equation

$$x^2 - y^2 = 2^3.$$