

**MATH250: FOUNDATIONS OF MATHEMATICS  
SPRING 2020**

**Homework 2 : due Wednesday, Feb. 5**

- Section 1.3: 3, 9, 12, 13.

**Additional problems:**

- (1) Write the negation of the following.
  - (a) If  $n$  is even, then  $n^2$  is even.
  - (b) If  $1 = 0$  and  $2 + 2 = 5$ , then  $n^2$  is even.
  - (c) If  $x$  and  $y$  are real numbers such that  $xy = 0$ , then  $x = 0$  or  $y = 0$ .
  - (d) For all real number  $x$ , if  $x \neq 0$ , then there is a real number  $y$  such that  $xy = 1$ .
  - (e) If there is a real number  $x$  such that  $2x = 1$ , then for all  $y$ ,  $y^2 < 0$ .
- (2) (Section 5.3, #2) Prove that  $n^2 - n$  is divisible by 2 for every integer  $n$ .
- (3) (Section 5.3, #5) Prove that if  $a \in \mathbb{Z}$ , then for every positive integer  $n$ ,  $a - 1$  divides  $a^n - 1$ .
- (4) Prove that the square of any integer of the form  $5k + 1$  for  $k \in \mathbb{Z}$  is of the form  $5k' + 1$  for some  $k' \in \mathbb{Z}$ .
- (5) Let  $n$  be a positive integer. Prove or disprove:  $n, n + 2, n + 4$  cannot all be prime.