MATH 175: ELEMENTARY FUNCTIONAL ANALYSIS (WINTER 2019)

Homework 9: due Wednesday, Mar. 13
(This is the last assignment.)

- Section 10.2: 10.2, 10.3.

Additional problems:

(1) Consider the Hilbert space $l^2(\mathbb{R})$. Let $T : l^2(\mathbb{R}) \to l^2(\mathbb{R})$ be defined by
$$T(x) = \left(x_1, \frac{x_2}{\sqrt{2}}, \frac{x_3}{\sqrt{3}}, \cdots, \frac{x_n}{\sqrt{n}}, \cdots\right).$$
Show that T is a compact operator but not Hilbert-Schmidt.

(2) Consider the Sturm-Liouville problem
$$f'' + \lambda f = 0, \quad 0 \le x \le L$$
$$f(0) = 0, \quad f'(L) = 0.$$
Find the eigenvalues and eigenfunctions explicitly.

(3) Consider the Sturm-Liouville problem
$$f'' + \lambda f = 0, \quad 0 \le x \le L$$
$$f(0) = 0, \quad f'(L) + hf(L) = 0$$
where $h > 0$ is a constant. Find the eigenfunctions explicitly. (The eigenvalues cannot be found explicitly. Draw a graph to demonstrate them.)