MATH 175: ELEMENTARY FUNCTIONAL ANALYSIS (WINTER 2019)

Homework 6: due Wednesday, Feb. 20

- Section 7.7: 7.1, 7.8, 7.9, 7.14, 7.15

Additional problem:

(1) Let F be a linear functional on a normed vector space X. Let $Ker(F)$ be the kernel of F. Show that F is continuous if and only if $Ker(F)$ is closed.
(Hint: first show that if F is unbounded, then there is a sequence $\{x_n\}_{n=1}^{\infty}$ such that $\|x_n\| \to 0$ as $n \to \infty$ and $F(x_n) = 1$. Then use linearity of F.)

(2) Let f be a continuous linear functional on a subspace Y of a Hilbert space X. Prove that f has a unique norm-preserving extension to a continuous linear functional on X.