MATH 131P: PRACTICE MIDTERM
(WINTER 2019)

Note: This is a closed book, no calculator exam. You have 50 min to complete the exam.

Problem 1: State the type of each of the following equations (linear or nonlinear, homogeneous or non-homogeneous, elliptic, hyperbolic or parabolic, order).

(1) $u_{xx} - u_{yy} = 0$, where $u = u(x, y)$.
(2) $u_t + uu_{xxx} = 0$, where $u = u(t, x)$.
(3) $u_t = u_{xx} + f(x, t)$, where $u = u(x, t)$.

Problem 2: Solve the equation ($\alpha > 0$)

$$\begin{gathered} u_t = \alpha^2 u_{xx}, \quad 0 < x < L, \quad 0 < t < \infty \\ u(0, t) = 1, \quad t > 0 \\ u(L, t) = 1, \quad t > 0 \\ u(x, 0) = 1 - 3\sin(2\pi x/L), \quad 0 \leq x \leq L. \end{gathered}$$

Problem 3: Consider the equation ($\alpha > 0$)

$$\begin{gathered} u_t = \alpha^2 u_{xx}, \quad 0 < x < L, \quad 0 < t < \infty \\ u_x(0, t) = 0, \quad t > 0 \\ u(L, t) = 0, \quad t > 0 \\ u(x, 0) = \phi(x), \quad 0 \leq x \leq L \end{gathered}$$

(1) Find the steady state solution.
(2) Solve the equation using separation of variables. (You need to find the coefficients of the expansion in terms of integrals but you do not need to evaluate the integrals.)

Problem 4: Solve the following equation using the Fourier transform ($\alpha > 0$)

$$\begin{gathered} u_t = \alpha^2 u_{xx} - u, \quad -\infty < x < \infty, 0 < t < \infty \\ u(x, 0) = \phi(x), \quad -\infty < x < \infty. \end{gathered}$$

Give the answer as a convolution in terms of ϕ. Find the solution explicitly when $\phi(x) = e^{-x^2}$.