MATH131P: PRACTICE FINAL EXAM (WINTER 2019)

Instructions: This is a closed book, no calculators exam. There are 8 problems. Write your solutions in blue book. You may solve the problems in any order, but please clearly indicate the number of problems. You must follow the instructions. Correct answer with wrong method will result in no credit! You have 150 minutes to complete the exam.

Problem 1: Solve the equation

$$u_x + u_t + tu = 0, \quad -\infty < x < \infty, 0 < t < \infty$$
$$u(x,0) = e^{-x^2}, \quad -\infty < x < \infty.$$

Problem 2: Find the solution of the exterior Neumann problem

$$\nabla^2 u = 0, \quad 1 < r < \infty, 0 \le \theta < 2\pi$$
$$\partial_r u(1,\theta) = \sin(\theta), \quad 0 \le \theta < 2\pi.$$

Problem 3: Solve the problem

$$u_t = u_{xx} + \sin(3\pi x), \quad 0 < x < 1, 0 < t < \infty$$
$$u(0,t) = 0,$$
$$u(1,t) = 1,$$
$$u(x,0) = \sin(\pi x), \quad 0 \le x \le 1.$$

Problem 4: Solve the semi-infinite string problem using the D'Alembert solution method

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < \infty, 0 < t < \infty$$
$$u_x(0,t) = 0, \quad 0 < t < \infty$$
$$u(x,0) = f(x),$$
$$u_t(x,0) = g(x).$$

Problem 5: Consider the Laplace equation $\nabla^2 u = 0$ on the rectangle $0 < x < a, 0 < y < b$ with boundary conditions

$$u(0,y) = 0, u(a,y) = y, u_y(x,0) = 0, u_y(x,b) = 0.$$

Find the solution using separation of variables. (You need to find the coefficients in the expansion of the solution explicitly, and you need to show all your work.)

Problem 6: Find the general solution of

$$u_{xx} - u_{xy} - 2u_{yy} = 0.$$

Problem 7: Solve the heat equation

$$u_t = \alpha^2 u_{xx}, \quad -\infty < x < \infty, 0 < t < \infty,$$
$$u(x,0) = \phi(x).$$

using Fourier transform. Express your solution in terms of a convolution.

Problem 8: Consider the problem

$$u_{tt} = \alpha^2 u_{xx}, \quad 0 < x < L, 0 < t < \infty$$
$$u(0,t) = T_1, \quad 0 < t < \infty$$
$$u(L,t) = T_2, \quad 0 < t < \infty$$
$$u(x,0) = \sin(\pi x/L), \quad 0 < x < L$$
$$u_t(x,0) = 0, \quad 0 < x < L.$$

(1) Transform the problem under the change of variables: $\xi = x/L, \tau = \alpha t/L$.
(2) Transform the problem under the change of variable

$$U(x,t) = \frac{u(x,t) - T_1}{T_2 - T_1}$$