# TWO EIGENVALUE PROBLEMS 

January 16, 2019

## Consider

$$
u^{\prime \prime}(t)+\lambda u(t)=0 .
$$

Given certain boundary conditions, you need to know how to find eigenvalues and eigenfunctions. There are two cases below which are frequently used in solving PDEs.

- Case (1):

$$
\begin{aligned}
& X^{\prime \prime}(x)+\lambda X(x)=0, \quad 0<x<L \\
& X(0)=0, \quad X(L)=0
\end{aligned}
$$

The eigenvalues and eigenfunctions are

$$
\begin{aligned}
& \lambda_{n}=\frac{n^{2} \pi^{2}}{L^{2}}, \quad n=1,2,3, \cdots \\
& X_{n}(x)=\sin \left(\frac{n \pi x}{L}\right), \quad n=1,2,3, \cdots
\end{aligned}
$$

- Case (2):

$$
\begin{aligned}
& X^{\prime \prime}(x)+\lambda X(x)=0, \quad 0<x<L \\
& X^{\prime}(0)=0, \quad X^{\prime}(L)=0
\end{aligned}
$$

The eigenvalues and eigenfunctions are

$$
\begin{aligned}
& \lambda_{0}=0, \quad \lambda_{n}=\frac{n^{2} \pi^{2}}{L^{2}}, \quad n=1,2,3, \cdots \\
& X_{0}(x)=1, \quad X_{n}(x)=\cos \left(\frac{n \pi x}{L}\right), \quad n=1,2,3, \cdots
\end{aligned}
$$

