## TWO EIGENVALUE PROBLEMS

January 16, 2019

Consider

$$u''(t) + \lambda u(t) = 0.$$

Given certain boundary conditions, you need to know how to find eigenvalues and eigenfunctions. There are two cases below which are frequently used in solving PDEs.

• Case (1):

$$X''(x) + \lambda X(x) = 0, \quad 0 < x < L,$$
  
 $X(0) = 0, \quad X(L) = 0.$ 

The eigenvalues and eigenfunctions are

$$\lambda_n = \frac{n^2 \pi^2}{L^2}, \quad n = 1, 2, 3, \cdots,$$
  
 $X_n(x) = \sin(\frac{n\pi x}{L}), \quad n = 1, 2, 3, \cdots.$ 

• Case (2):

$$\begin{aligned} X''(x) + \lambda X(x) &= 0, \quad 0 < x < L, \\ X'(0) &= 0, \quad X'(L) = 0. \end{aligned}$$

The eigenvalues and eigenfunctions are

$$\lambda_0 = 0, \quad \lambda_n = \frac{n^2 \pi^2}{L^2}, \quad n = 1, 2, 3, \cdots,$$
  
 $X_0(x) = 1, \quad X_n(x) = \cos(\frac{n\pi x}{L}), \quad n = 1, 2, 3, \cdots.$