MATH 126C SUMMER 2018
MIDTERM 2 REVIEW

July 29, 2018

Preparation:

- The exam is on Thursday, August 2nd in TA session:
  10:50am -11:50am BNS Room 117
- You should bring a Ti-30x IIS Calculator.
- You can bring one hand-written 8.5 by 11 inch page of notes (double-sided).
- The midterm covers Section 14.1, 14.3, 14.4, 14.7;
  15.1-15.4 and Section 10.3 (polar coordinates.)
- To prepare for the midterm, please review all homework problems and look at the exams in the department archive, starting from the most recent ones.

Important Topics:

(1) Functions of two variables $f(x, y)$: be able to find and graph the domain; know basics on level curves and contour maps.

(2) Partial derivatives: know how to compute first order partial derivatives $f_x(x, y)$ and $f_y(x, y)$; know their geometric meaning (i.e. slope of tangent lines in the x or y direnction.); know how to compute second order derivatives $f_{xx}, f_{yy}, f_{xy}, f_{yx}$ and know Clairaut's theorem: $f_{xy} = f_{yx}$

(3) Know how to find the equation of tangent plane of the surface $z = f(x, y)$ at given point:

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b).$$

know how to find and use linear approximation of $f(x, y)$.

(4) Be able to find critical points of $f(x, y)$ (that is to solve (x, y) from $f_x = 0, f_y = 0$); Know the second order derivative test and be able to use it to find local max, min and saddle points;

(5) know how to find the absolute max and min on a region: First, find critical points inside the region. Then, over each boundary, substitution the equation for the boundary into the surface to get a one variable function. Find the absolue max/min

of the one variable function over each boundary. At last, evaluate $f(x;y)$ at all the critical points inside the region and the critical numbers and endpoints (corners) on each boundary to find the largest and smallest value.

(6) Understand the meaning of double integral $\int\int_D f(x,y)dA$: that is the (signed) volume of solid between the surface $z = f(x,y)$ and $xy-$plane, in particular, area of D is $\int\int_D 1dA$.

(7) Know how to compute double integral using iterated integrals:
- for rectangle $R = [a,b] \times [c,d]$:
$$\int\int_R f(x,y)dA = \int_a^b \int_c^d f(x,y)dydx = \int_c^d \int_a^b f(x,y)dxdy$$
- for type I region $D = \{(x,y) : a \le x \le b, \quad g_1(x) \le y \le g_2(x)\}$
$$\int\int_D f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dydx$$
- for type II region $D = \{(x,y) : c \le y \le d, \quad h_1(y) \le x \le h_2(y)\}$
$$\int\int_D f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dxdy$$

(8) Know polar coordinates and conversion between Cartesian and polar coordinates; know how to compute double integrals in polar coordinates:
- for polar rectangle $R = \{(r,\theta) : a \le r \le b, \alpha \le \theta \le \beta\}$:
$$\int\int_R f(x,y)dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)rdrd\theta$$
- for region $D = \{(r,\theta) : \alpha \le \theta \le \beta, \quad h_1(\theta) \le r \le h_2(\theta)\}$
$$\int\int_D f(x,y)dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)rdrd\theta$$

(9) Application of double integrals: if $\rho(x,y)$ is the density of region D, know the formula for total mass
$$M = \int\int_D \rho(x,y)dA$$
and the formula for center of mass $(\bar{x}, \bar{y})$:
$$\bar{x} = \frac{\int\int_D x\rho(x,y)dA}{M}, \quad \bar{y} = \frac{\int\int_D y\rho(x,y)dA}{M}.$$