Math 112z, Fall 2019
Practice Midterm 3

Name: _____

Student ID Number: _____

- There are 6 pages of questions. Make sure your exam contains all these questions.
- This is a closed book, closed note, no calculator exam. You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- If you need more room, use the backs of the pages and indicate clearly that you have done so.
- Raise your hand if you have a question.
- Remember the **Honor Code**. Avoid suspicion of cheating by keeping your eyes on your paper and clearly showing your work on each problem!
- The problems are not ordered according to their difficulties, so please take a look at all problems and do not waste too much time on one problem. Budget your time wisely.
- You have 75 minutes to complete the exam.

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GOOD LUCK!

1. (20 pts) Determine whether the series are absolutely convergent, conditionally convergent or divergent.

   (a) $\sum_{n=1}^{\infty}(-1)^n \cos(1/n^2)$.

   (b) $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+4}$

   (c) $\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$

2. (10 pts) Use the Maclaurin series $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ to evaluate the integral

$$\int \sin(x^2) dx.$$

3. (20 pts) Consider the power series:

$$\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln n}$$

(a) Find the radius of convergence.

(b) Find the interval of convergence.

4. (20 pts) Use the power series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, |x| < 1$ for this problem.

(a) Find a power series for $f(x) = \frac{2}{3-5x}$.

(b) Find the radius of convergence for the power series in part (a).

(c) Find the power series representation of $f(x) = \frac{1}{(1-x)^2}$.

(d) Find a power series representation for $f(x) = (\frac{x}{1-x})^2$

5. (15 pts) Find a Taylor series for $f(x) = x^{-2}$ about $a = 1$.

6. (15 pts) Consider $f(x) = 2\cos x$.

   (a) Find a 4th degree Taylor polynomial approximation $T_4(x)$ for $f(x)$ about $a = \pi/3$.

   (b) Use Taylor's inequality to estimate the accuracy of the approximation $f(x) \simeq T_4(x)$ when x lies in the interval $0 \leq x \leq 2\pi/3$.