Math 112z, Fall 2019
Practice Midterm 2

Name: 

Student ID Number: 

- There are 6 pages of questions. Make sure your exam contains all these questions.
- This is a closed book, closed note, no calculator exam. You must show your work on all problems. The correct answer with no supporting work may result in no credit.

- **Put a box around your FINAL ANSWER for each problem and cross out any work that you don’t want to be graded.**
- If you need more room, use the backs of the pages and indicate clearly that you have done so.
- Raise your hand if you have a question.
- Remember the **Honor Code**. Avoid suspicion of cheating by keeping your eyes on your paper and clearly showing your work on each problem!
- The problems are not ordered according to their difficulties, so please take a look at all problems and do not waste too much time on one problem. Budget your time wisely.
- You have 75 minutes to complete the exam.

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| **Total** | **100** |

GOOD LUCK!
1. (15 pts) Find all solutions of the equation

\[(x^2 + 1)y' = xy\]
2. (15 pts) Solve the initial value problem

\[ xy' - y = x \ln x, \quad y(1) = 2. \]

Note \( x > 0 \) in this problem.
3. (20 pts) Newton’s law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and the surrounding temperature. Let $T(t)$ be the temperature of the object and $T_s$ be the surrounding temperature. We get

$$\frac{dT}{dt} = k(T - T_s)$$

where $k$ is a constant. Suppose that the temperature of the object is 200°F in the beginning and 1 minute later, it has cooled down to 190°F in a room at 70°F. Find the time when the temperature of the object becomes 150°F.
4. (15 pts) Determine whether the sequence converges or diverges. If it converges, find the limit.

(a) \( a_n = (1 + 2/n)^n \).

(b) \( a_n = 2^{-n} \cos(n\pi) \).
5. (15 pts) Determine whether the series converges or diverges. If it converges, find the sum.

(a) \[ \sum_{n=1}^{\infty} \frac{1}{1 + e^{-n}} \]

(b) \[ \sum_{n=2}^{\infty} \frac{1}{n \ln n} \]

(c) \[ \sum_{n=1}^{\infty} \ln(1 + 1/n) \]
6. (20 pts) The following two problems are independent of each other.

(a) Find constant $c$ such that
\[ \sum_{n=0}^{\infty} e^{cn} = 10. \]

(b) Consider the sequence $a_n = \frac{3^n}{n!}$. Determine whether it converges or diverges.

(Note: the original problem was for $a_n = \frac{(-3)^n}{n!}$.)