QUALIFYING EXAM SYLLABUS

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1. Algebraic number theory

Chapters I and II in Neukirch [Neu13] and the statements of Class Field Theory, as in Bjorn Poonen's notes [Poo12]. Specifically:

1.1. **Number Fields.** Dedekind domains, norm and trace, ring of integers, discriminant and different, Galois extensions, prime factorization in extension fields, the ideal class group, finiteness of the class number, Dirichlet's unit theorem, quadratic and cyclotomic extensions, decomposition and inertia groups.

1.2. **Local fields.** Hensel's lemma and Henselian fields, completions and valuations, extensions of valuations, unramified, tamely ramified, and totally ramified extensions.

1.3. **Class field theory.** Adeles and Ideles. Statements of class field theory, including Artin Reciprocity and the Chebotarev Density theorem.

References:

- Neukirch's book [Neu13].
- Milne's notes [Mil20].
- Sutherland's notes [Sut21].
- Poonen's notes [Poo12].

2. Algebraic geometry

Sections II.1-8 and III.1-5 in [Har13] and the statements in sections IV.1-3 [Har13].

2.1. **Schemes.** Affine schemes, irreducibility, reducedness, noetherianity, fibered products.

2.2. **Morphisms.** Separability, properness (valuative criteria), finite morphisms, finite type morphisms, closed immersions, projective morphisms.

2.3. **Sheaves of modules.** Quasi-coherent sheaves, coherent sheaves, twisting sheaf.

2.4. **Divisors.** Weil divisors, Cartier divisors, invertible sheaves, the Picard group, linear systems, morphisms to projective space.

2.5. **Differentials.** Differentials, sheaves of differentials.

2.6. **Cohomology.** Cohomology of sheaves, Cech cohomology, Cohomology of projective spaces, Grothendieck's vanishing theorem, Serre's criteria for affineness, Serre's duality (statement).

Date: June 3, 2022.

2.7. **Curves.** Statements of the Riemann-Roch theorem and the Riemann-Hurwitz theorem.

References:

- Hartshorne's book [Har13].
- Vakil's book [Vak18].
- Bosch's book [Bos13].

References

[Bos13] Siegfried Bosch, *Algebraic Geometry and Commutative Algebra*, 1 ed., Springer, London, 2013.

[Har13] Robin Hartshorne, *Algebraic Geometry*, vol. 52, Springer Science & Business Media, 2013.

[Mil20] James S. Milne, *Algebraic number theory (v3.08)*, 2020, Available at www.jmilne.org/math/, p. 166.

[Neu13] Jürgen Neukirch, *Algebraic number theory*, vol. 322, Springer Science & Business Media, 2013.

[Poo12] Bjorn Poonen, *A Brief Summary of the Statements of Class Field theory*, 2012.

[Sut21] Andrew Sutherland, *18.785 - Number Theory I Lecture notes*, https://math.mit.edu/classes/18.785/2021fa/lectures.html, 2021.

[Vak18] Ravi Vakil, *The rising sea: Foundations of algebraic geometry*, preprint (2018).