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# Results and Problems On Chorded Cycles: A Survey

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**Abstract** A *chord* of a cycle $C$ is an edge between two non-consecutive vertices of the cycle. A cycle $C$ in a graph $G$ is *chorded* if the vertex set of $C$ induces at least one chord. In 1961 Posa formulated a natural question: What conditions imply a graph contains a chorded cycle? In this paper, we survey results and problems that relate to Posa's question on chorded cycles in graphs. These include sufficient conditions for a chorded cycle to exist, or sets of chorded cycles exist, or cycles with multiple chords exist, or chorded cycles with additional properties exist.

**Keywords** Chord · cycle · chorded cycle · pancyclic.

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## 1 Introduction

The study of cycles in graphs is a rich and an important area. One natural question is to study cycles satisfying certain other conditions. These conditions include properties like sets of independent cycles, cycles containing specified vertices or edges, cycle length, and cycles spanning the vertex set. Over the last few years answers to a new question on cycles with additional properties has gained interest. A *chord* of a cycle is an edge between two non-consecutive

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vertices of the cycle. We say a cycle is *chorded* if it contains at least one chord, and the cycle is *doubly chorded* if it contains at least two chords, etc.

In 1961 Posa [62] asked a very natural question:

*Question 1* What conditions imply a graph contains a chorded cycle?

Posa (see [62], solution prob. 10.2, p 376) supplied one answer by showing that a graph on $n$ vertices with at least $2n-3$ edges contains a chorded cycle. However, real interest in this problem took years to develop. Recently there has been a considerable increase in results on this question. These new results have shown that Posa's question is an important and natural one, as these results extend the general theory on cycles in graphs, and our overall understanding of substructures in graphs.

In this paper we will survey results that provide answers to Posa's question. We will also look at more particular questions and problems for further study in regard to his question. We will not address areas like chordal graphs, as this topic has already been well studied.

All graphs are simple. For a graph $G$, denote the vertex and edge sets of $G$ as $V(G)$ and $E(G)$, respectively, Let $d(v)$ denote the degree of the vertex $v$. We denote by $G = (V, X, Y)$ a bipartite graph with vertex set $V$ and partite sets $X$ and $Y$. A cycle of length $\ell$ is called an *$\ell$-cycle*. For $u \in V(G)$, the set of neighbors of $u$ in $G$ is denoted by $N_G(u)$, and $N_G[v] = N(v) \cup \{v\}$. Note that if the graph is clear we will simple use $N(v)$ and $N[v]$. When the graph in question is clear we may simply use $N(u)$. Let $H$ be a subgraph of $G$, and let $S \subseteq V(G)$. For $u \in V(G) - V(H)$, we denote $N_H(u) = N_G(u) \cap V(H)$ and $d_H(u) = |N_H(u)|$. For $X \subseteq V(G)$, let $d_H(X) = \sum_{x \in X} d_H(x)$. For $u \in V(G) - S$, $N_S(u) = N_G(u) \cap S$. Furthermore, $N_G(S) = \cup_{w \in S} N_G(w)$ and $N_H(S) = N_G(S) \cap V(H)$. Let $A, B$ be two vertex-disjoint subgraphs of $G$. Then $N_G(A) = N_G(V(A))$ and $N_B(A) = N_G(A) \cap V(B)$. If $S = \{u\}$, then we write $G - u$ for $G - S$. For two disjoint graphs $G_1$ and $G_2$, the graph $G_1 \cup G_2$ denotes the disjoint union of $G_1$ and $G_2$. Further, we denote by $G_1 \vee G_2$ the *join* of $G_1$ and $G_2$, that is, $G_1 \vee G_2$ has vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$ together with all edges from $V(G_1)$ to $V(G_2)$. A *2-factor* in a graph $G$ is a collection of disjoint cycles whose vertices span $V(G)$. A graph $G$ is *pancyclic* if it contains cycles of each length from 3 to $|V(G)|$ and $G$ is *$k$-pancyclic* if it contains cycles of each length from $k$ to $|V(G)|$. Let $\sigma_k(G) = \min\{\sum_{i=1}^{k} \deg(x_i)\}$ where the minimum is taken over all independent sets of vertices $x_1, x_2, \ldots, x_k$ in $G$. Let $\sigma_k(G)\ \infty$ if the independence number of $G$ is less than $k$. When this degree sum condition holds for any $k \geq 2$ we