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# 2-Factors in Claw-free Graphs

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July 30, 1999

**Abstract**

We consider the question of the range of the number of cycles possible in a 2-factor of a 2-connected claw-free graph with sufficiently high minimum degree. (By claw-free we mean the graph has no induced $K_{1,3}$.) In particular, we show that for such a graph $G$ of order $n \geq 51$ with $\delta(G) \geq \frac{n-2}{3}$, $G$ contains a 2-factor with exactly $k$-cycles, for $1 \leq k \leq \frac{n-24}{3}$. We also show that this result is sharp in the sense that if we if we lower $\delta(G)$, we cannot obtain the full range of values for $k$.

## 1 Introduction

The question of determining when a graph contains a 2-factor (a 2-regular spanning subgraph) has long been an important one in graph theory. Many results deal with hamiltonian graphs, that is, graphs $G$ containing a cycle that spans the vertex set $V(G)$. (See [4].) One special class of graphs that has drawn considerable interest are the claw-free graphs. Such graphs contain no induced subgraph isomorphic to the complete bipartite graph $K_{1,3}$.

In particular, the following was shown in [5].

**Theorem 1** *If $G$ is a 2-connected $K_{1,3}$-free graph of order $n$ with $\delta(G) \geq \frac{n-2}{3}$, then $G$ is hamiltonian.*

We can see that this result is sharp by considering the following nonhamiltonian graph $G$ on $n = 3m$ vertices. Let $V(G) = A_1 \cup A_2 \cup A_3$ such that $|A_i| = m$ and $\langle A_i \rangle \cong K_m$ and let $x_i, y_i \in A_i$, $x_i \neq y_i$ for $i = 1, 2, 3$ and so that $\langle x_1, x_2, x_3 \rangle \cong \langle y_1, y_2, y_3 \rangle \cong K_3$. Clearly, the minimum degree of $G$ is $m - 1 = \frac{n-3}{3}$.

Recently the question of determining the number of cycles possible in a 2-factor of a given 2-connected graph satisfying certain degree conditions has been considered in [2].

The purpose of this paper is to investigate this question for 2-connected claw-free graphs. In particular, we will extend Theorem 1 by showing that the same minimum degree condition implies that $G$ contains a 2-factor with exactly $k$-cycles for $1 \leq k \leq \frac{n-24}{3}$.

We will let $\langle S \rangle_G$ denote the subgraph of $G$ induced by $S$ a subset of $V(G)$. For $A, B \subset V(G)$, $e_G(A, B)$ denotes the number of edges in $G$ with one vertex in $A$ and the other in $B$. For $H \subset G$ we will sometimes write $e_G(A, H)$ as shorthand for $e_G(A, V(H))$. The independence number of a graph will be denoted by $\alpha(G)$. For a cycle $C$, we will denote by $\overrightarrow{C}$ the cycle under some orientation and $\overleftarrow{C}$ will denote the cycle under the opposite orientation. For a vertex, $a$, on a cycle with some orientation, $\overrightarrow{C}$, we define $a^+$ and $a^-$ to be the immediate successor and predecessor respectively of $a$ on $C$ with respect to this orientation. Also, for a collection of vertex disjoint cycles $S$ each with some orientation, we define $N_S^+(a)$ to be the set $\{a^+ | a \in (N(a) \cap V(S))\}$. Let $I = \overrightarrow{a_1, a_2, ..., a_k}$ where the $a_i$'s are consecutive vertices on a cycle. Then $l(I) = k$, the length of the segment of the cycle. For terms not defined here, see [3].

## 2 Main Result

In this section we will prove the theorem. However, first we prove the following proposition which gives sufficient conditions for the existence of $k$ disjoint triangles and will lay the foundation for the proof of the theorem.

**Proposition 1** *Let $G$ be a claw-free graph of order $n$, let $k$ be an integer, and let $c \geq 0$. If $n > 3k + 6 - f(k, c)$ where $f(1, 1) = f(2, 0) = 0$ and $f(k, c) = \frac{9c-9}{k+c-2}$ for all other values of $k$ and $c$ and $\delta(G) \geq max\{k + c, 3\}$ then $G$ contains $k$ disjoint triangles.*

**Proof:** If $\delta(G) \geq 3$, then $n \geq 4$ and, since $G$ is claw-free, $G$ must contain at least one triangle. Choose $m$ disjoint triangles in $G$, say $T_1, T_2, ..., T_m$, so that $m$ is as large as possible. Since $G$ is claw-free and $\delta(G) \geq 3$, we know $m \geq 1$. Assume $m < k$. Let

$$A = \bigcup_{i=1}^{m} V(T_i)$$

and $H = G - A$.

If $\Delta(H) \geq 3$, say $\deg_H a \geq 3$ for some $a \in V(H)$, then since $G$ is claw-free, $b_1 b_2 \in E(H)$ for some $b_1, b_2 \in N_H(a)$ and $\{a, b_1, b_2\}$ forms a triangle. This contradicts the maximality of $m$. Therefore, $\Delta(H) \leq 2$.

**Claim:** For each $x \in A$, $|N_G(x) \cap V(H)| \leq 3$.

**Proof:** Assume $|N_G(x) \cap V(H)| \geq 4$ for some $x \in A$. Let $x \in V(T_i)$ and $V(T_i) = \{x, y, z\}$. Let $a_1, a_2, a_3, a_4$ be distinct neighbors of $x$ in $H$.

If $N_G(a_1) \cap \{a_2, a_3, a_4\} = \emptyset$, then since $x$ and $\{a_1, a_2, a_3\}$ do not form a claw, without loss of generality, $a_2a_3 \in E(G)$. We apply the same argument to $x$ and $\{a_1, a_2, a_4\}$ and $\{a_1, a_3, a_4\}$, and we have $a_2a_4 \in E(G)$ and $a_3a_4 \in E(G)$. But then $\{a_2, a_3, a_4\}$ forms a triangle, which contradicts the maximality of $m$. Therefore, $N_G(a_1) \cap \{a_2, a_3, a_4\} \neq \emptyset$. Similarly, we have $\deg_{\langle a_1, a_2, a_3, a_4 \rangle_G} a_i \geq 1$ for each $i$, $1 \leq i \leq 4$. Since $\Delta(H) \leq 2$, we know $\langle a_1, a_2, a_3, a_4 \rangle_H = \langle a_1, a_2, a_3, a_4 \rangle_G$ must contain two independent edges. Thus, without loss of generality, we may assume $a_1a_2, a_3a_4 \in E(G)$.

Consider the subgraph induced by $F = \langle \{a_1, a_2, a_3, a_4, y, z\} \rangle_G$. We want to show that $F$ must contain $K_3 \cup K_2$ as a subgraph because the existence of such a subgraph in $F$ implies that $\langle F \cup \{x\} \rangle$ contains two independent triangles which contradicts the maximality of $A$.

In order to show that $F$ must contain $K_3 \cup K_2$, we first observe that to avoid a claw centered at $x$, $F$ cannot contain 3 independent vertices. Let $S_1 = \langle \{a_1, a_2\} \rangle$, $S_2 = \langle \{a_3, a_4\} \rangle$, and $S_3 = \langle \{y, z\} \rangle$. Then there are 8 independent 3-sets of vertices in $S_1 \cup S_2 \cup S_3$. Note that the addition of any edge to $S_1 \cup S_2 \cup S_3$ can destroy at most two of the 8 independent triples of vertices. Thus, $F$ must have at least 4 more edges than $S_1 \cup S_2 \cup S_3$. Without loss of generality, we can assume there are two edges between $S_1$ and $S_2$. If these two edges share an endvertex, then $F$ contains $K_3 \cup K_2$. Thus, we may assume they are independent. By symmetry, we may further assume that they are $a_1a_3$ and $a_2a_4$. Moreover, again to avoid $K_3 \cup K_2$ in $F$, we may assume $\{a_1a_4, a_2a_3\} \cap E(G) = \emptyset$. Then by considering the triple $\{a_1, a_4, y\}$ we can, without loss of generality, assume $F$ contains the edge $a_1y$. But, by the same argument, the triple $\{a_2, a_3, y\}$ forces the edge $a_2y$ or $a_3y$, and therefore $F$ contains a triangle and an independent edge. This contradicts the maximality of $A$ and the claim follows. □

Since $\Delta(H) \leq 2$ and $\delta(G) \geq k + c$, we have $e_G(x, A) \geq k + c - 2$, for each $x \in V(H)$. Thus, $e_G(H, A) \geq (k + c - 2)(n - 3m)$. On the other hand, $e_G(u, H) \leq 3$ for each $u \in A$ which implies $e_G(A, H) \leq 3|A| = 9m$. Therefore, $(k + c - 2)(n - 3m) \leq 9m$. Thus, $(k + c - 2)n \leq (3k + 3c + 3)m$. Then, using the fact that we assumed $m \leq k - 1$, we find $n \leq \frac{3k^2+3ck-(3c+3)}{k+c-2} = 3k + 6 - \frac{9c-9}{k+c-2}$. This contradicts the assumption and completes the proof. □

**Theorem 2** *Let $G$ be a 2-connected, claw-free graph of order $n \geq 51$ with $\delta(G) \geq \frac{1}{3}(n-2)$. Then for each $k$ with $1 \leq k \leq \frac{n-24}{3}$, $G$ has a 2-factor with exactly $k$ components.*

**Proof:** By the assumption $n \geq 3k+24$ and $\delta(G) \geq \frac{n-2}{3} \geq \frac{3k+22}{3} \geq k+1$. Therefore, by Proposition 1, $G$ has $k$ disjoint cycles $C_1, C_2, ..., C_k$. Choose $C_1, ..., C_k$ such that $\sum_{i=1}^{k} |V(C_i)|$ is as large as possible. Let $D = \bigcup_{i=1}^{k} V(C_i)$ and assume $D \neq V(G)$. Let $H = G - D$ and let $x \in V(H)$.

**Claim 1:** $|V(H) \geq 4$.

**Proof:** Let $h = |V(H)|$ and assume $h \leq 3$.

Since $h \leq 3$, $|D| \geq n-3 \geq 3k+21$. Thus, there exists some cycle, say $C_i$, such that $|V(C_i)| \geq 4$. Let $x \in V(H)$ and let $|N_G(x) \cap V(C_i)| = t$, say $N_G(x) \cap C_i = \{a_1, ..., a_t\}$. We may assume $a_1, ..., a_t$ appear in consecutive order along some orientation of $C_i$. Let $I_j = a_j \overrightarrow{C_i} a_{j+1}$ for $1 \leq j \leq t-1$ and let $I_t = a_t \overrightarrow{C_i} a_1$. If $l(I_j) = 1$ for some $1 \leq j \leq t$, then $a_{j+1} = a_j^+$. Let $C_i' = a_{j+1} \overrightarrow{C_i} a_j x a_{j+1}$ and $C_j' = C_j$ for all $j \neq i$. Then $\{C_1', ..., C_k'\}$ is a disjoint collection of cycles of larger total order, a contradiction. Therefore, $l(I_j) \geq 2$ for each $j$, $1 \leq j \leq t$.

Since $G$ is claw-free, this implies $a_j^- a_j^+ \in E(G)$ for each $j$, $1 \leq j \leq t$. If $l(I_j) = 2$, then $a_j^{++} = a_{j+1}$. Let $C_i' = x a_{j+1} \overrightarrow{C_i} a_j^- a_j^+ a_j x$ and $C_j' = C_j$ for all $j \neq i$. If $l(I_j) = 3$, then $a_j^{+++} = a_{j+1}$. Let $C_i' = x a_{j+1} a_{j+1}^- a_{j+1}^+ \overrightarrow{C} a_j^- a_j^+ a_j x$ and $C_j' = C_j$ for all $j \neq i$. In either case, the collection $\{C_1', ... C_k'\}$ forms a set of independent cycles of larger order, a contradiction.

Therefore, $l(I_j) \geq 4$ for each $j$, $1 \leq j \leq t$. This implies $|V(C_i)| = \sum_{j=1}^{t} l(I_j) \geq 4t$ or $|N_G(x) \cap V(C_i)| \leq \frac{1}{4}|V(C_i)|$ for all $C_i$ such that $|V(C_i)| \geq 4$. Note that $x$ has at most one adjacency to every 3-cycle in the collection $C_1, \cdots, C_k$.

We may assume $|V(C_1)| = |V(C_2)| = ... = |V(C_s)| = 3$ and $|V(C_i)| \geq 4$ for $s+1 \leq i \leq k$. Then,

$$\begin{aligned}
\frac{n-2}{3} &\leq e(x, D) + \deg_H x \\
&\leq (h-1) + s + \frac{1}{4} \sum_{i=s+1}^{k} |V(C_i)| = (h-1) + \frac{|D| + s}{4} \\
&= \frac{n-h+s}{4},
\end{aligned}$$

which implies $n < 3s + 9h - 4$. Since $s < k$ and $h < 3$, we have $n < 3k + 23$. This contradicts the assumption. Consequently, we know $|V(H)| > 4$. □

**Claim 2:** For each $y \in V(H) - \{x\}$, $\deg_{H-x} y \geq 2$.

**Proof:** Assume $\deg_{H-x} y \leq 1$ for some $y \in V(H) - \{x\}$. As in Claim 1, we count the number of edges from $y$ to $D$ observing that $y$ can have at most one adjacency to a 3-cycle and $y$ is adjacent to at most one out of every four vertices on cycles of length 4 or more.

We may assume $|V(C_1)| = |V(C_2)| = ... = |V(C_s)| = 3$ and $|V(C_i)| \geq 4$ for $s+1 \leq i \leq k$. Then $e(y, D) \leq s + \frac{1}{4} \sum_{i=s+1}^{k} |V(C_i)| = s + \frac{1}{4}(|D| - 3s) = \frac{1}{4}|D| + \frac{1}{4}s$. Therefore,

$$\begin{aligned} \frac{n-2}{3} &\leq \deg_H y + \deg_D y \leq \deg_H y + e(y, D) \\ &\leq 1 + \deg_{H-x} y + \frac{1}{4}|D| + \frac{1}{4}s \\ &\leq 1 + \deg_{H-x} y + \frac{1}{4}(n-4) + \frac{1}{4}k \\ &\leq 1 + \deg_{H-x} y + \frac{1}{4}(n-4) + \frac{n-24}{12}. \end{aligned}$$

Thus, $\deg_{H-x} y \geq 2$. □

By Claims 1 and 2, we know that for every $x \in V(H)$, $H - x$ contains a cycle, call it $C_x$.

**Claim 3:** For every $x \in H$, the set $N_D^+(x)$ is independent.

**Proof:** Assume, to the contrary, $a_1^+ a_2^+ \in E(G)$ for some $a_1, a_2 \in N_D(x)$. If $a_1$ and $a_2$ lie in the same cycle of $D$, say $C_i$. Then we increase the total order of $D$ by replacing $C_i$ by $C_i' = a_1^+ \overrightarrow{C_i} a_2 x a_1 \overleftarrow{C_i} a_2^+ a_1^+$. If $a_1$ and $a_2$ lie in different cycles of $D$, we may assume without loss of generality $a_i \in V(C_i), i = 1, 2$. Then let $C_1' = C_x$, $C_2' = x a_1 \overleftarrow{C_1} a_1^+ a_2^+ \overrightarrow{C_2} a_2 x$ and for $j \neq 1, 2$ let $C_j' = C_j$. Then the collection $\{C_1', ...C_k'\}$ forms a set of $k$ disjoint cycles of larger total order, a contradiction. □

**Claim 4:** Since $G$ is a claw-free graph of order $n$, then $\alpha(G) \leq \frac{2n}{\delta(G)+2}$.

**Proof:** Let $S$ be a largest independent set in $G$. For each $x \in V(G) - S$, we have $e_G(x, S) \leq 2$ since G is claw-free. Therefore, $e_G(S, V(G) - S) \leq 2|V(G) - S| = 2(n - \alpha(G))$. On the other hand, since $S$ is independent, we know $e_G(S, V(G) - S) = \sum_{x \in S} \deg_G x \geq \delta(G)|S| = \delta(G)\alpha(G)$.

Therefore, we have $2(n - \alpha(G)) \geq \delta(G)\alpha(G)$. Solving this inequality for the independence number and we get $\alpha(G) \leq \frac{2n}{\delta(G)+2}$. □

By Claims 3 and 4, for each $x \in V(H)$ we have that

$$|N_D[x]| = |N_D^+(x) \cup \{x\}| \le \alpha(G) \le \frac{2n}{\delta(G)+2} \le \frac{2n}{\frac{n-2}{3}+2} < 6.$$

Therefore, $|N_D(x)| \le 4$ and we have $\deg_H x \ge \frac{n-14}{3}$.

Let $P$ be a longest path in $H$ and let $x$ be one of its end vertices. Then $N_H(x) \subseteq V(P)$ or a longer path is possible. Therefore, if we choose $y \in N_H(x)$ so that $x\overrightarrow{P}y$ is as long as possible, we form a cycle $C = x\overrightarrow{P}yx$ with $N_H(x) \subseteq V(C)$. This implies $|V(C)| \ge \deg_H x + 1 \ge \frac{n-14}{3} + 1 = \frac{n-11}{3}$. Then by the maximality of $D$, we know $|V(C_i)| \ge \frac{n-11}{3}$, for all $1 \le i \le k$.

**Claim 5:** The number of independent cycles, $k$, is 2.

**Proof:** Assume $k \ge 3$. Then $n = |V(G)| \ge |V(C)| + |V(C_1)| + |V(C_2)| + |V(C_3)| \ge 4(\frac{n-11}{3})$. This forces $n \le 44$, a contradiction. □

Since $C_1$ and $C_2$ each have at least $\frac{n-11}{3}$ vertices, we know $|V(H)| \le n - |V(C_1)| - |V(C_2)| \le \frac{n+22}{3}$.

**Claim 6:** The subgraph $H$ is hamiltonian connected.

**Proof:** If $H$ is not hamiltonian-connected by [6],

$$\frac{n-14}{3} \le \delta(H) \le \frac{1}{2}|V(H)| \le \frac{n+22}{6}.$$

This forces $n \le 50$, a contradiction. □

In particular, $H$ has a hamiltonian cycle, say $C_0$. By the maximality of $D$, we know $|V(C_0)| \le |V(C_i)|$ for $i = 1, 2$. Thus, $|V(C_0)| \le \frac{1}{3}n$.

Since $G$ is 2-connected, there exist at least two independent edges between $C_0$ and $C_1 \cup C_2$.

**Claim 7:** There do not exist two independent edges from $C_0$ to $C_i$, for $i = 1, 2$.

**Proof:** Without loss of generality, let $i = 1$. Assume there are two independent edges, say $a_1b_1$ and $a_2b_2$ between $C_0$ and $C_1$ (where $a_1, a_2 \in C_0$, $b_1, b_2 \in C_1$). Without loss of generality, we may assume $l(b_1\overrightarrow{C_1}b_2) \geq \frac{1}{2}|V(C_1)|$. Since $\{a_2\overrightarrow{P}a_1b_1\overrightarrow{C_1}b_2a_2, C_2\}$ forms a set of disjoint cycles where $P$ is a hamiltonian $a_1, a_2$-path in $H$, we know $l(b_2\overrightarrow{C_1}b_1) \geq |V(C_0)| + 1 \geq \delta(H) + 2 \geq \frac{n-8}{3}$. Then $|V(C_1)| \geq 2l(b_2\overrightarrow{C}b_1) \geq \frac{2n-16}{3}$. Therefore,

$$n = |V(C_0)| + |V(C_1)| + |V(C_2)| \geq 2\left(\frac{n-11}{3}\right) + \frac{2n-16}{3} = \frac{4n-38}{3}.$$

This forces $n \leq 38$ which is a contradiction. □

Therefore we may assume $a_1b_1, a_2b_2 \in E(G)$ where $a_1, a_2 \in V(C_0)$, $a_1 \neq a_2$, $b_1 \in V(C_1)$, and $b_2 \in V(C_2)$. As a consequence of Claim 7 and 2-connectivity, we know there exists an edge $d_1d_2 \in E(G)$ such that $d_1 \in V(C_1) - b_1$ and $d_2 \in V(C_2)$.

Let $x \in H - \{a_1, a_2\}$. (Since $|V(H)| = |V(C_0)| \geq \frac{n-11}{3}$ we know such an $x$ exists.) Then by Claim 6, $N_{C_1 \cup C_2}(x) \subset \{b_1, b_2\}$. Therefore, $\deg_H x \geq \frac{n-2}{3} - 2 = \frac{n-8}{3}$, and hence $|V(C_0)| \geq \frac{n-5}{3}$.

**Claim 8:** The graph $H - \{a_1, a_2\}$ has a triangle $T$ and $H - V(T)$ is hamiltonian-connected.
**Proof:** Let $H' = H - \{a_1, a_2\}$ and assume $\delta(H') \leq \frac{|V(H')|}{2}$. Since $\delta(H') \geq \delta(H) - 2 \geq \frac{n-8}{3} - 2 \geq \frac{n-14}{3}$ and $|V(H')| \leq \frac{n}{3} - 2 = \frac{n-6}{3}$, we get $\frac{n-14}{3} \leq \frac{1}{2}\left(\frac{n-6}{3}\right)$. This forces $n \leq 18$, a contradiction.

Thus $\delta(H') \geq \frac{|V(H')|+1}{2}$ and $|V(H')| \geq \frac{n-5}{3} - 2 \geq 3$, which implies by [1] that $H'$ is pancyclic. Thus $H'$ has a triangle $T$. Let $H'' = H - V(T)$. Then $|V(H'')| = |V(H)| - 3 \geq \frac{n}{3} - 3 = \frac{n-9}{3}$ and $\delta(H'') \geq \frac{n-14}{3} - 3 \geq \frac{n-23}{3}$. Therefore, since $n \geq 40$, $\delta(H'') > \frac{1}{2}|V(H'')|$. Hence, by [6] $H''$ is hamiltonian connected. □

First, suppose $d_2 \neq b_2$. We may assume $l(d_1\overrightarrow{C_1}b_1) \leq \frac{1}{2}(|V(C_1)|)$ and $l(b_2\overrightarrow{C_2}d_2) \leq \frac{1}{2}(|V(C_2)|)$. By the maximality of $C_1$ and $C_2$ and the fact that $G$ is claw-free, $b_1^+b_1^-, b_2^+b_2^- \in E(G)$. Let $C' = a_1b_1b_1^-b_1^+\overrightarrow{C_1}d_1d_2\overrightarrow{C_2}b_2^-b_2^+b_2a_2Pa_1$, where $P$ is a hamiltonian $a_1a_2$-path in $H - T$. Since $C'$ and $T$ are disjoint cycles, $l(d_1^+\overrightarrow{C_1}b_1^{--}) + l(b_2^{++}\overrightarrow{C_2}d_2^-) + 2 \geq |V(H)|$. Thus $\frac{|V(C_1)|+|V(C_2)|}{2} - 4 \geq |V(H)| \geq \frac{n-5}{3}$, which implies that $|V(C_1)| + |V(C_2)| \geq \frac{2n+14}{3}$. Since $|V(H)| = |V(C_0)| \geq \frac{n-5}{3}$, we have $n = |V(H)| + |V(C_1)| + |V(C_2)| \geq \frac{3n+9}{3} = n + 3$, a contradiction. Therefore, we know $d_2 = b_2$

which implies that there cannot be three independent edges between the cycles $C, C_1$, and $C_2$.

Since $G$ is 2-connected, there exists an edge $b_2'u$ from $C_2 - \{b_2\}$ to $C_0 \cup C_1$

**Case 1:** We consider the case where $u \in C_0$. If $u \neq a_1$ the three edges $a_1b_1, d_1b_2$, and $b_2'u$ are independent, a contradiction. Thus, $u = a_1$. But now the two edges $a_2b_2$ and $a_1b_2'$ between $C_0$ and $C_2$ are independent. This contradicts Claim 7.

**Case 2:** We consider the case where $u \in C_1$. If $u \neq b_1$, then the three edges $a_1b_1, ub_2'$, and $a_2b_2$ are independent, a contradiction. If $u = b_1$, consider $b_1$ and $\{a_1, b_1^+, b_2'\}$. We know $b_2'b_1^+ \notin E(G)$ because $u = b_1$. By Claim 7, $a_1b_2' \notin E(G)$. If $a_1b_1^+ \in E(G)$, then the three edges $a_1b_1^+, b_1b_2'$ and $a_2b_2$ are independent, a contradiction. Thus, $\langle b_1, b_1^+, a_1, b_2' \rangle_G$ is a claw, a contradiction.

Hence, in all cases we reach a contradiction, and the result is proved. □

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