On the Structure of Ryjáček-closed, $\{K_{1,3}, N_2\}$-free Graphs

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1 Introduction

Given a simple graph G and a family $\mathcal{F}$ of connected graphs, we say that G is $\mathcal{F}$-free if G contains no induced subgraph that is isomorphic to a graph in $\mathcal{F}$. The graphs considered in this paper will be graphs that are $\{K_{1,3}, N_2\}$-free, where the graphs $K_{1,3}$ and N_2 are as shown in Figure 1. The graph $K_{1,3}$ is often called the *claw*. The graph N_2 has also been referred to as the *Eiffel*, denoted by E (see the survey [2] by Faudree, Flandrin and Ryjáček).

Another common forbidden subgraph is the *net*, or N, and it is simply the graph consisting of a triangle with one pendant leaf attached at each of its three vertices. Shepherd [4] has investigated graphs that are $\{K_{1,3}, N\}$-free, and the results of our paper can be viewed as an extension of some of his work.

If v is a vertex of a graph G, and if S is a subset of the vertex set of G, we will use the following notation regarding neighborhoods:

$$N(v) = \{x \in V(G) : vx \in E(G)\},$$

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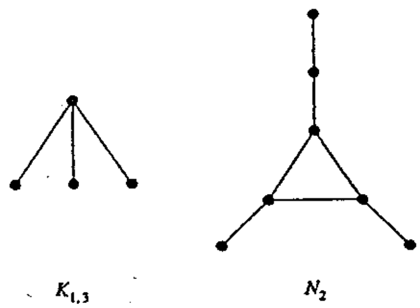

Figure 1: The forbidden pair.

$$N_S(v) = \{x \in S : vx \in E(G)\}.$$

If v is a vertex of a claw-free graph G, then the graph induced by the neighbors of v, denoted $< N(v) >$, must have at most two components. We say that G is *Ryjáček-closed* if for every vertex v, each component of $< N(v) >$ is complete. This term originates from some ideas developed in a recent paper of Ryjáček [3].

The graphs that we consider in this paper are connected, Ryjáček-closed, $\{K_{1,3}, N_2\}$-free graphs. Our motivation in studying the structure of the graphs in this fairly restrictive class stemmed from our investigation of the hamiltonian properties of a particular subset of this class, and the results of that investigation will appear separately.

2 Preliminaries

Let T be a minimum cut set of the graph G, and let v be a vertex of T. Let $S = T \setminus \{v\}$. Since G is connected and claw-free, v must be a cut vertex of the graph $G \setminus S$, and there must be exactly two (nonempty) components of the graph $G \setminus S \setminus \{v\}$ $(= G \setminus T)$. Call these two components A and B. Within the component A, we will construct "distance sets" (with respect to v) as follows: Let $A_i = \{x \in A : d(x, v) = i\}$. We construct similar sets on the B side. Since G is finite, there must be integers n and m such that A_n, B_m are both nonempty, while A_{n+1}, B_{m+1} are both empty. The situation in G is depicted in Figure 2. For notational convenience, we define A_0 and B_0 to be $\{v\}$, and A_{-i} to be B_i.

We now make several preliminary observations.

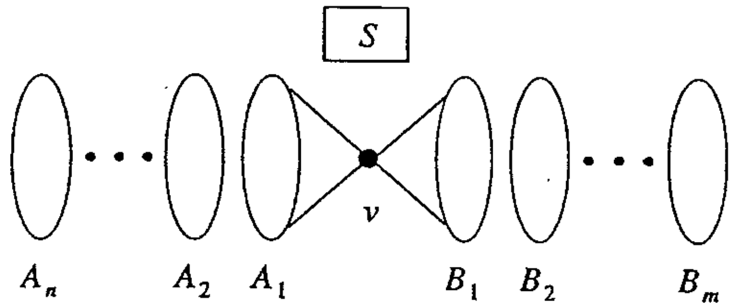

Figure 2:

(i) Each vertex in the set S must be adjacent to some vertex in A and to some vertex in B. This follows immediately from the fact that T is a minimum cut set.

(ii) There are no edges of the form ab where $a \in A$ and $b \in B$. This is due to the nature of the two sets A and B.

(iii) If $a_i \in A_i$ is adjacent to $a_j \in A_j$ then either $i = j$ or $|i - j| = 1$. This says that there are no "jumping edges," and this is due to the nature of the distance sets themselves.

(iv) Each of $< A_1 >$ and $< B_1 >$ are complete. For if a_1 and a_1' were nonadjacent vertices of A_1, then $< \{v, a_1, a_1', b\} >$ would be an induced claw, where b is any vertex in B_1. Thus A_1 induces a complete graph, and, similarly, so does B_1.

(v) Each vertex of A_1 and B_1 is adjacent to v. Also, given a vertex $a_i \in A_i (i > 1)$, we see that a_i must be adjacent to at least one vertex in A_{i-1}. Also, a_i may or may not be adjacent to a vertex in A_{i+1}. If a_i is adjacent to a vertex in A_{i+1}, we will call a_i a *continuer*. Otherwise we will call a_i a *noncontinuer*. The terms have similar meanings on the B side.

3 The Distance Sets

It was noted earlier that A_1 and B_1 both induce complete subgraphs. In this section we will consider the other distance sets in the context of the various possibilities for the values of n and m. We assume without loss of generality that $n \geq m$. We also note that Propositions 1, 2, and 4 are true without the assumption that G is Ryjáček-closed.

Proposition 1 *If $n+m \geq 5$ then each A_i and B_j induces a complete subgraph.*

Proof:

Case 1: Suppose $m = 1$ (and thus $n \geq 4$).

We first claim that each of $A_3, A_4, \ldots, A_n$ induces a complete subgraph. Suppose not, and let k be the least integer greater than or equal to 3 such that $< A_k >$ is not complete. Suppose that a_k and a'_k are nonadjacent vertices of A_k. If a_k and a'_k have a common neighbor in A_{k-1}, say a_{k-1}, then $< \{a_{k-1}, a_k, a'_k, a_{k-2}\} >$ is an induced claw where $a_{k-2} \in N_{A_{k-2}}(a_{k-1})$. Thus a_k and a'_k can have no common neighbors in A_{k-1}.

From a previous note we know that a_k, a'_k must each have at least one neighbor in A_{k-1}. Let $a_{k-1}, a'_{k-1} \in A_{k-1}$ be neighbors of a_k, a'_k respectively.

Suppose that a_{k-1} and a'_{k-1} are adjacent, and let $a_{k-2} \in A_{k-2}$ be a neighbor of a_{k-1}. Since $< \{a_{k-1}, a_k, a'_{k-1}, a_{k-2}\} >$ is a potential claw, the edge $a_{k-2}a'_{k-1}$ must be present. But then if $a_{k-3} \in A_{k-3}$ is a neighbor of a_{k-2}, and $a_{k-4} \in A_{k-4}$ is a neighbor of a_{k-3}, we have an induced N_2: $< \{a_k, a'_k, a_{k-1}, a'_{k-1}, a_{k-2}, a_{k-3}, a_{k-4}\} >$ (see Figure 3).

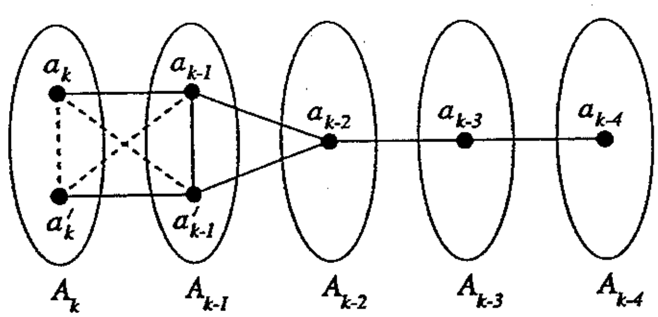

Figure 3:

Therefore it cannot be that a_{k-1} and a'_{k-1} are adjacent. Thus it must be that $k = 3$ (and a_2, a'_2 are not adjacent). The vertices a_2 and a'_2 must then have distinct neighbors in A_1, say a_1 and a'_1 respectively. But then if b is any vertex of B_1, we have an induced N_2: $< \{a_3, a_2, a'_2, a_1, a'_1, v, b\} >$. Therefore it must be that $A_3, A_4, \ldots, A_n$ each induce complete subgraphs.

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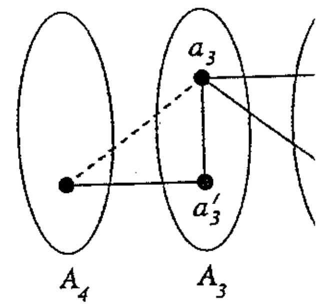

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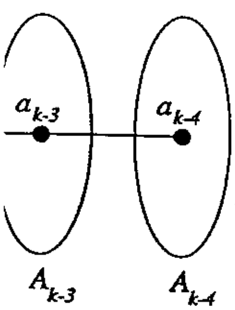

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Now consider A_2. Suppose that a_2 and a'_2 are nonadjacent vertices in A_2. If a_2 is a continuer (say $a_3 \in A_3$ is adjacent to a_2), then $< \{a_3, a_2, a'_2, a_1, a'_1, v, b\} >$ is a potential induced N_2. Thus the edge $a_3a'_2$ must be present. If a_3 were a continuer (say to $a_4 \in A_4$, then $< \{a_3, a_4, a_2, a'_2\} >$ would be an induced claw, so a_3 must not be a continuer. Since $n \geq 4$, we know that A_3 must contain a continuer, say a'_3 (see Figure 4).

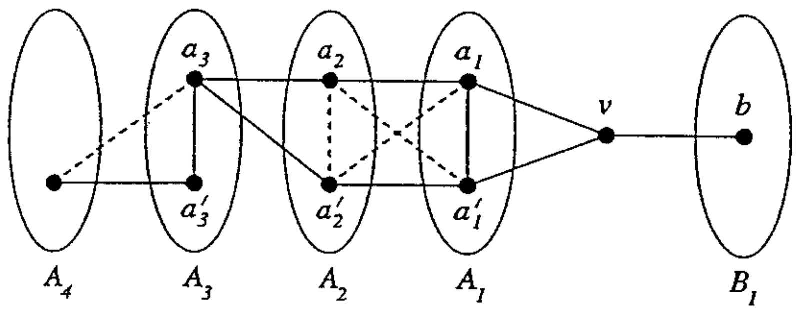


Figure 4:

If either a_2 or a'_2 were adjacent to a'_3, we would have an induced N_2 by our previous argument. So a'_3 must be nonadjacent to both a_2 and a'_2. But then $< \{a_3, a'_3, a_2, a'_2\} >$ is an induced claw.

Therefore we can assume that neither a_2 nor a'_2 is a continuer. There must be a continuer in A_2, so let one be a''_2, and let $a_3 \in N_{A_3}(a''_2)$. If either of a_2 or a'_2 were nonadjacent to the continuer a''_2, we would have an induced N_2 by the previous argument. Thus both a_2 and a'_2 are adjacent to a''_2. But then $< \{a''_2, a_2, a'_2, a_3\} >$ is an induced claw. Thus, having reached contradictions in all cases, we can conclude that a_2 must be adjacent to a'_2, and hence that $< A_2 >$ is complete.

This concludes the argument for this case.

Case 2: Suppose that $m \geq 2$ (and thus $n \geq 3$).

We first show that each of $A_2, A_3, \ldots, A_n$ induce complete subgraphs. Suppose not, and let k be the least integer such that $< A_k >$ is not complete (clearly $k \geq 2$). Let a_k and a'_k be nonadjacent vertices in A_k. Again, they must have distinct neighbors in A_{k-1}, or else we would have an induced claw. Let $a_{k-1}, a'_{k-1} \in A_{k-1}$ be neighbors of a_k, a'_k, respectively. Let $a_{k-2} \in A_{k-2}$ be a neighbor of a_{k-1}. Since G is claw-free, it must also be that a_{k-2} is a neighbor of a'_{k-1}. Let $a_{k-3} \in N_{A_{k-3}}(a_{k-2})$ and $a_{k-4} \in N_{A_{k-4}}(a_{k-3})$. Then we see that

$< \{a_k, a'_k, a_{k-1}, a'_{k-1}, a_{k-2}, a_{k-3}, a_{k-4}\} >$ is an induced N_2, a contradiction. Therefore it must be that each of $A_2, A_3, \ldots, A_n$ induces a complete subgraph. By a similar argument, we can conclude that each of $B_2, \ldots, B_m$ induces a complete subgraph.

This concludes Case 2, and also the proof of the proposition. ∎

The next three propostions concern the structure of the distance sets in the case where $n + m \leq 4$.

Proposition 2 *If $n+m \leq 4$ and $n = m$, then each A_i and B_j induces a complete subgraph.*

Proof: Clearly the only possibilities are $n = m = 1$ and $n = m = 2$. The result has been established already for $n = m = 1$, and so we assume that $n = m = 2$. Suppose a_2 and a'_2 are nonadjacent vertices of A_2. Let $a_1, a'_1 \in A_1$ be neighbors of a_2, a'_2, respectively. Let b_2 be a vertex of B_2, and let $b_1 \in B_1$ be a neighbor of b_2. Then $< \{a_2, a'_2, a_1, a'_1, v, b_1, b_2\} >$ is an induced N_2, which is a contradiction. Thus $< A_2 >$ must be complete, and similarly, so must $< B_2 >$. ∎

Proposition 3 *If $n = 2$ and $m = 1$, then the vertices of A_2 can be partitioned into sets $A_2^1, A_2^2, \ldots, A_2^p$ such that*

(i) there exist p distinct vertices in A_1, say $a_1^1, \ldots, a_1^p$ such that for each i, $N_{A_1}(x) = \{a_1^i\}$ for each $x \in A_2^i$;

(ii) $< A_2^i >$ is complete for each i.

Proof: Consider a vertex $a_2 \in A_2$. Clearly it has a neighbor in A_1, say a_1. If a'_1 $(\neq a_1)$ is also a neighbor of a_2 in A_1, then a_2, a_1, and v are in the same connected component of $N(a'_1)$. Since G is Ryjáček-closed, this implies that a_2 is adjacent to v, which is a contradiction. Thus, a_2 has exactly one neighbor in A_1.

Letting $a_1^1, a_1^2, \ldots, a_1^p$ be the continuers in A_1, we can partition the vertices in A_2 into the p sets $A_2^1, \ldots, A_2^p$ where $A_2^i = \{x \in A_2 : xa_1^i \in E(G)\}$.

Now for an arbitrary i, let x, y be vertices of A_2^i. If x and y are nonadjacent, then $< \{a_1^i, x, y, v\} >$ would be an induced claw. Thus $< A_2^i >$ must be complete. ∎

The following lemma is a straightforward corollary to a theorem of Chvátal and Erdös [1].

Lemma 1 *If the independence number of a graph G is 2, then G is either the union of two complete graphs, or it contains a hamiltonian path.*

Proposition 4 *If $n = 3$ and $m = 1$, then $< A_3 >$ is complete, and $< A_2 >$ is either a complete subgraph, two complete subgraphs, or it contains a spanning path.*

Proof: Suppose $< A_3 >$ is not complete, and let $a_3, a_3' \in A_3$ be nonadjacent. Since G is claw-free, it must be that these two vertices have distinct neighbors in A_2. Let $a_2 \in N_{A_2}(a_3)$ and $a_2' \in N_{A_2}(a_3')$. Also let $a_1 \in N_{A_1}(a_2)$ and $b \in B_1$. If $a_2 a_2' \in E(G)$. then $< \{a_3, a_3', a_2, a_2', a_1, v, b\} >$ is an induced N_2, a contradiction. So, suppose that $a_2 a_2' \notin E(G)$. Since G is claw-free, a_2 and a_2' do not share a neighbor in A_1, so let $a_1' \in N_{A_1}(a_2')$. G being claw-free also implies that $a_3 a_2' \notin E(G)$. But then $< \{a_3, a_2, a_1, a_1', a_2', v, b\} >$ is an induced N_2, also a contradiction. Thus $< A_3 >$ must be complete.

Now consider $< A_2 >$. Suppose that the independence number of $< A_2 >$ is greater than 2. Then there must exist three vertices $a_2, a_2', a_2'' \in A_2$ that are mutually nonadjacent. As can be seen from earlier arguments, no two of these vertices can share a neighbor in A_1, so let $a_1, a_1', a_1'' \in A_1$ be distinct neighbors of a_2, a_2', a_2'', respectively. Suppose that a_2 is a continuer, and say that $a_3 \in N_{A_3}(a_2)$. If a_3 is adjacent to both a_2' and a_2'', then $< \{a_3, a_2, a_2', a_2''\} >$ is an induced claw. Thus a_3 must be nonadjacent to at least one of a_2' and a_2''. Suppose, without loss, that a_3 is nonadjacent to a_2'. Then $< \{a_3, a_2, a_2', a_1, a_1', v, b\} >$ is an induced N_2 where b is any vertex of B_1. Thus a_2 cannot be a continuer, and, similarly, neither can a_2' or a_2''.

A_2 must contain at least one continuer, though, so let one be a_2''', and let $a_3''' \in N_{A_3}(a_2''')$ (see Figure 5). Since a_2''' is a continuer, it must be adjacent to at least two of a_2, a_2', a_2'' (or else the previous contradiction would be obtained). Say that a_2''' is adjacent to a_2 and a_2'. Then $< \{a_2''', a_2, a_2', a_3'''\} >$ is an induced claw, which is a contradiction.

Therefore the independence number of $< A_2 >$ is at most 2. From the lemma, we see that $< A_2 >$ is either a complete subgraph, the union of two complete subgraphs, or it contains a spanning path. ∎

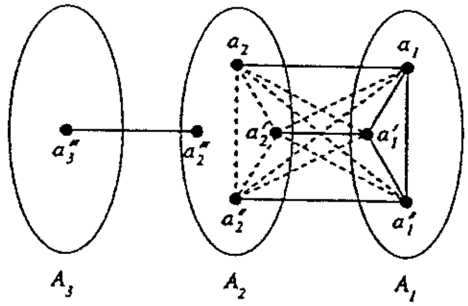


Figure 5:

We conclude this section by summarizing our findings regarding the distance sets. In the case where $n = 2, m = 1$, we found that $< A_2 >$ was composed of complete subgraphs, each one having a unique neighbor in A_1. In the case where $n = 3, m = 1$, we found that $< A_2 >$ was either a clique, two cliques, or traceable, and the other distance sets all induced complete subgraphs. In all other cases we found that each of the distance sets induced a complete subgraph.

4 The Cut Set Vertices

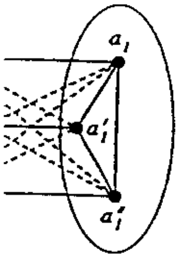
We begin our discussion of the vertices of S with an observation. As noted earlier, each vertex of S must be adjacent to something in A and to something in B. Note that if $s \in S$ is adjacent to $a_i \in A_i$ and $a_j \in A_j (i \neq j)$, then it must be that $|i-j| = 1$. Otherwise, $< \{s, a_i, a_j, b\} >$ would be an induced claw, where $b \in N_B(s)$. Therefore we can see that each $s \in S$ is incident with at most two of the sets $A_1, A_2, \ldots, A_n$ (and by a similar argument, at most two of the sets $B_1, \ldots, B_m$).

Divide the vertices of S into two sets as follows: let S_v be the vertices of S which are adjacent to v, and let S_0 be the vertices of S which are not adjacent to v.

Proposition 5 *The vertices of S_v can be partitioned into two disjoint sets, S_{vA} and S_{vB}, such that*

(i) each vertex of S_{vA} is adjacent to all of A_1 and none of B_1;

(ii) each vertex of S_{vB} is adjacent to all of B_1 and none of A_1;



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(iii) each of $< S_{vA} >$ and $< S_{vB} >$ is complete.

Proof: First note that if $s \in S_v$ is adjacent to $a_i \in A_i$ and $b_j \in B_j$,then at least one of i or j must be 1. Otherwise, $< \{s, a_i, v, b_j\} >$ is an induced claw. Furthermore, if both i and j equal 1, then we have that a_i, v, b_j are all in the same connected component of $N(s)$, which implies (since G is Ryjáček closed) that a_i is adjacent to b_j, which is a contradiction. Thus it must be that s is incident with exactly one of A_1 or B_1. We can therefore split the vertices of S_v into two disjoint sets: let S_{vA} be those vertices of S_v which are incident with A_1, and let S_{vB} be those vertices of S_v which are incident with B_1.

Now, let s be an arbitrary vertex of S_{vA}, and suppose it is adjacent to $a_1 \in A_1$. Let a_1' be any other vertex of A_1. Since s, a_1, and a_1' are all in the same connected component of $N(v)$, and since G is Ryjáček-closed, it must be that s is adjacent to a_1'. We see then that s must be adjacent to all of A_1, and we can see (after similar arguments for S_{vB}) that properties (i) and (ii) in the statement of the proposition are satisfied.

Let s and s' be vertices of S_{vA}. If a_1 is an arbitrary vertex of A_1, then we see that s, s', and v are in the same connected component of $N(a_1)$, so it must be that s and s' are adjacent. We conclude then that $< S_{vA} >$ is complete, and similarly that $< S_{vB} >$ is complete. ∎

Proposition 6 *For $n \geq 3$, if $s \in S_0$ is nonadjacent to all vertices of $A_1 \cup A_2$, then s must be adjacent to a vertex in A_n.*

Proof: The proposition is trivial for $n = 3$, and thus we consider the case where $n \geq 4$. Suppose that s is nonadjacent to all vertices in $A_1 \cup A_2 \cup A_n$. Let k be the least positive integer such that s is incident with A_k, and let $a_k \in N_{A_k}(s)$. Further, let $a_{k-1} \in N_{A_{k-1}}(a_k)$. If j is the least positive integer such that s is incident with B_j, then let $b_j \in N_{B_j}(s)$ and let $b_{j-1} \in N_{B_{j-1}}(b_j)$ (see Figure 6). It is possible that $b_{j-1} = v$.

Case 1: Suppose that a_k is a continuer. If x is any vertex of $N_{A_{k+1}}(a_k)$, then the edge sx must be present (otherwise $< \{a_k, a_{k-1}, s, x\} >$ is an induced claw). This also implies that $k + 1 < n$.

If a_k is adjacent to any continuer in A_{k+1}, say a_{k+1}, and if $a_{k+2} \in N_{A_{k+2}}(a_{k+1})$, then $< \{a_{k+2}, a_{k+1}, a_k, a_{k-1}, s, b_j, b_{j-1}\} >$ is an induced

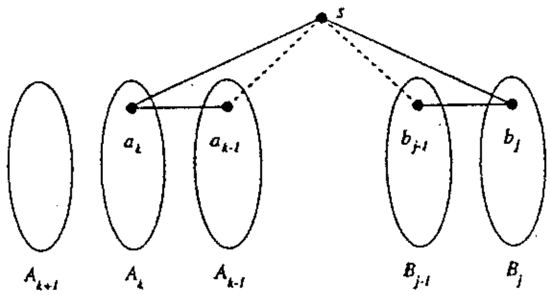

Figure 6:

N_2, a contradiction. So suppose that a_k is adjacent only to noncontinuers in A_{k+1}, let $a_{k+1} \in N_{A_{k+1}}(a_k)$, and let a'_{k+1} be a continuer in A_{k+1}. Consider the graph $< \{a_{k+1}, a'_{k+1}, s, a_k, a_{k-1}, b_j, b_{j-1}\} >$. Since this is a potential N_2, it must be that the edge sa'_{k+1} is present. But then s, a'_{k+1}, and a_k are all in the same connected component of $N(a_{k+1})$, and this implies that a_k is adjacent to the continuer a'_{k+1}, which is a contradiction to our assumption. Therefore, we can conclude that a_k cannot be a continuer.

Case 2: Suppose that a_k is a noncontinuer. Let a'_k be a continuer in A_k, let $a_{k+1} \in N_{A_{k+1}}(a'_k)$, and let $a_{k-2} \in N_{A_{k-2}}(a_{k-1})$. If $sa'_k \in E(G)$, then Case 1 applies with a'_k instead of a_k. So assume that $sa'_k \notin E(G)$. Now, G being claw-free implies both that $a'_k a_{k-1} \in E(G)$ and that $sa_{k+1} \notin E(G)$. But then the subgraph $< \{a_{k+1}, a'_k, a_k, a_{k-1}, a_{k-2}, s, b_j\} >$ is an induced N_2, which is a contradiction.

Having reached contradictions in both cases, we see that it is not possible for $s \in S_0$ to be simultaneously nonadjacent to all of the vertices in A_1, A_2, and A_n. ∎

In summary of the results of this section, we recall that the vertices in S which are adjacent to v can be partitioned into two subsets, each of which induces a complete subgraph which is incident with exactly one of A_1 and B_1. The vertices of S which are not adjacent to v are fairly limited as to the distance sets in A and B with which they can be incident.

5 Final Remarks

We would like to express o
many others responsible fo
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consideration, and helpful

References

[1] Chvátal, V. and Erdő
crete Math. 2 (1972),

[2] Faudree, R., Flandrin
survey. *Discrete Math*

[3] Ryjáček, Z., On a clos
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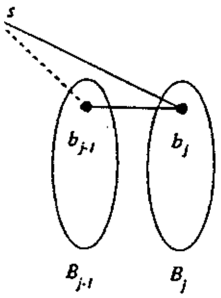
[4] Shepherd, F. B., Ha
Theory Ser. B 53 (19



is adjacent only to noncontin-
et a'_{k+1} be a continuer in A_{k+1}.
$a_{k-1}, b_j, b_{j-1}\}$ >. Since this is
e sa'_{k+1} is present. But then
ected component of $N(a_{k+1})$,
he continuer a'_{k+1}, which is a
fore, we can conclude that a_k

r. Let a'_k be a continuer in A_k,
$_{-2}(a_{k-1})$. If $sa'_k \in E(G)$, then
ssume that $sa'_k \notin E(G)$. Now,
$_{-1} \in E(G)$ and that $sa_{k+1} \notin$
$_k, a_k, a_{k-1}, a_{k-2}, s, b_j\}$ > is an

h cases, we see that it is not
ly nonadjacent to all of the
∎

on, we recall that the vertices
itioned into two subsets, each
hich is incident with exactly
ich are not adjacent to v are
and B with which they can

5 Final Remarks

We would like to express our appreciation to the directors and to the many others responsible for organizing the Eighth International Conference. The presentations and discussions were stimulating, and the fellowship was enjoyable. We also thank the referee for his/her time, consideration, and helpful suggestions.

References

[1] Chvátal, V. and Erdõs, P., A note on hamiltonian circuits. *Discrete Math.* 2 (1972), 111-113.

[2] Faudree, R., Flandrin, E. and Ryjáček, Z., Claw-free graphs – A survey. *Discrete Math.* 164 (1997), 87-147.

[3] Ryjáček, Z., On a closure concept in claw-free graphs. *J. Combin. Theory Ser. B* (to appear).

[4] Shepherd, F. B., Hamiltonicity in claw-free graphs. *J. Combin. Theory Ser. B* 53 (1991), 173-194.