Math 535 Combinatorics I, Fall 2016

Meeting time and location: MW 2:30-3:45p in MSC E406

Instructor:	Hao Huang
Office:	MSC E432
Office hours:	By appointment.
Email:	hao.huang@emory.edu

Textbook

Most of the topics covered in this course appear in the books and surveys listed below, especially the first one.

- *Linear algebra methods in combinatorics*, by László Babai and Peter Frankl, Department of Computer Science, University of Chicago, preliminary version, 1992.
- *Combinatorial Nullstellensatz*, by Noga Alon. Download from `http://www.tau.ac.il/~nogaa/PDFS/null2.pdf`
- *Tools from higher algebra*, by Noga Alon. Download from `http://www.tau.ac.il/~nogaa/PDFS/tools1.pdf`.
- *Polynomial Methods in Combinatorics*, by Larry Guth. University Lecture Series, American Mathematical Society.

Course Description

Combinatorics is a fundamental branch of mathematics and its problems naturally arises in many areas of pure and applied mathematics, notably algebra, probability, number theory, geometry and complexity theory. In the past, many of the basic combinatorial problems have been considered in isolation, and often ad-hoc proofs are given mainly based on ingenuity and detailed reasoning. However in the last few decades, its study has experienced fast development and most of the recent results rely on deep and well-developed theories. One of the most general and powerful techniques that plays an important role is the algebraic method.

The purpose of this one-semester-long course is to provide a fairly complete and rigorous treatment of the fundamental theory and the applications of algebraic methods in combinatorics, and hence it serves as a good first course for graduate students and higher level undergraduate students in mathematics, computer science, or engineering that are interested in combinatorics or graph theory. The prerequisites for this course are MATH 250 (Foundations of Mathematics), MATH 321 (Abstract Vector Spaces), or equivalent courses. This means good comprehension of basic algebraic motions and the ability to understand and write a rigorous mathematical proof.

For some of the applications mentioned in this class, backgrounds in graph theory, number theory, algebraic geometry and Fourier analysis will be very helpful. But I will assume no knowledge of these and make sure the lectures are self-contained.

Below is a list of topics that will be covered in this course:

- Basic rank and dimension arguments.

- The polynomial method. Set systems with restricted intersections.
- The Combinatorial Nullstellensatz and Chevalley-Warning Theorem.
- Spectral graph theory.
- Tensor product methods.
- Convexity arguments: Radon's Lemma and Helly's Theorem.

These algebraic techniques are used for a wide array of applications, not just in combinatorics and graph theory, but also in discrete geometry, additive number theory, and algorithms. This include some classical result like the solution of Kakeya problem in finite fields, the construction of counterexamples to Borsuk's conjecture, explicit constructions of Ramsey graphs, the Berlekamp-Welch Algorithm, and the very recent breakthrough by Croot, Lev, Pach, Ellenberg and Gijswijt on the capset problem.

Grading and Homework

During the semester, there will be three homework assignments. Students are required to submit them to get a final grade.

The homework assignments will be posted and updated on the class webpage, and announced in the classes or sent by emails. Every student must write their own solution independently. ****Please note that no late homework will be accepted or credited.****.