Math 346, HW2 Solution

2.4.4

Assume that x_{ij} is the number of units sent from Warehouse i to Outlet j. Then the constraints for each warehouse and outlet are:

$$
\begin{aligned}
&x_{11} + x_{12} + x_{13} + x_{14} \leq 100 \\
&x_{21} + x_{22} + x_{23} + x_{24} \leq 150 \\
&x_{31} + x_{32} + x_{33} + x_{34} \leq 300 \\
&x_{11} + x_{21} + x_{31} \geq 120 \\
&x_{12} + x_{22} + x_{32} \geq 120 \\
&x_{13} + x_{23} + x_{33} \geq 120 \\
&x_{14} + x_{24} + x_{34} \geq 120
\end{aligned}
$$

Moreover, outlet 2 cannot receive more units from warehouse 1 than from warehouse 2, and outlet 4 must receive at least half of its suuply from warehouse 3. This gives us two additional constraints:

$$
\begin{aligned}
&x_{12} \leq x_{22} \\
&x_{34} \geq (x_{14} + x_{24} + x_{34})/2
\end{aligned}
$$

To compute the objective function, note that the storage charge is \$6 per unit at Warehouse 1 and 2, and \$12 per unit in Warehouse 3. Therefore the storage cost is $6(100 - x_{11} - x_{12} - x_{13} - x_{14}) + 6(150 - x_{21} - x_{22} - x_{23} - x_{24}) + 12(300 - x_{31} - x_{32} - x_{33} - x_{34})$. And the shipping cost is $12x_{11} + 15x_{12} + 10x_{13} + 25x_{14} + 10x_{21} + 19x_{22} + 11x_{23} + 30x_{24} + 21x_{31} + 30x_{32} + 18x_{33} + 40x_{34}$. The sum of shipping cost and storage cost would be our objective function. And the constraints are listed above, together with the nonnegative constraint that all $x_{ij} \geq 0$.

2.4.6

Suppose we ship x_{ij} units from Source i to Destination j for $(i, j) \neq (2, 1)$. x_{21} is the number of units below 20 sent from Source 2 to Destination 1, and y is the number of units above 20 sent from Source 2 to Destination 1.

$$\begin{array}{ll}\text{minimize} & 8x_{11}+17x_{12}+19x_{13}+21x_{22}+22x_{23}+10x_{21}+13y\\ \text{subject to} & x_{11}+x_{12}+x_{13}\le 80\\ & x_{21}+y+x_{22}+x_{23}\le 80\\ & x_{11}+x_{21}+y\ge 50\\ & x_{12}+x_{22}\ge 50\\ & x_{13}+x_{23}\ge 50\\ & x_{ij}\ge 0, x_{21}\le 20, y\ge 0\end{array}$$

2.5.3

We assume that in the i-th month, the dealer buys B_i units from the manufacturer, sells to the student S_i units, and store T_i units.

$$\begin{array}{ll}\text{Maximize} & (90S_1+110S_2+105S_3)-(60B_1+65B_2+68B_3)-7(T_1+T_2+T_3)\\ \text{subject to} & 25+B_1=S_1+T_1\\ & T_1+B_2=S_2+T_2\\ & T_2+B_3=S_3+T_3\\ & 0\le B_i\le 65, 0\le S_i\le 100, 0\le T_i\le 45.\end{array}$$

2.6.9

Suppose in the i-th month, the shop produces D_i units of differentials, and store S_i units. Each month the shop uses B_i hours below 400 hrs, and O_i hours above 400 hrs.

$$\begin{array}{ll}\text{minimize} & 3(12D_1+17D_2+25D_3+26D_4)+18(B_1+B_2+B_3+B_4)\\ & +26(O_1+O_2+O_3+O_4)+10(S_1+S_2+S_3+S_4)\\ \text{subject to} & D_1=225+S_1\\ & S_1+D_2=225+S_2\\ & S_2+D_3=225+S_3\\ & S_3+D_4=225+S_4\\ & 2D_i\le B_i+O_i\\ & D_i\ge 0, S_i\ge 0, 0\le B_i\le 400, 0\le O_i\le 150.\end{array}$$

3.1.3

(e) First we introduce two new variables x_3 and x_4 to convert the inequalities into equalities:

$$\begin{array}{ll}\text{Minimize} & 6x_1+x_2\\ \text{subject to} & -5x_1+8x_2+x_3=80\\ & x_1+2x_1-x_4=4\\ & x_1\le 10, x_2, x_3, x_4\ge 0.\end{array}$$

To replace x_1 by a nonnegative variable. There are two ways: the first is to let $x_1 = 10 - x_1'$, then $x_1' = 10 - x_1 \geq 0$. Replace all the x_1 by $10 - x_1'$ in the LP, we get:

$$\begin{array}{ll} \text{Minimize} & 6(10 - x_1') + x_2 \\ \text{subject to} & -5(10 - x_1') + 8x_2 + x_3 = 80 \\ & (10 - x_1') + 2x_1 - x_4 = 4 \\ & x_1', x_2, x_3, x_4 \geq 0. \end{array}$$

We could rearrange the terms so that the constant terms are on the right:

$$\begin{array}{ll} \text{Minimize} & 60 - 6x_1' + x_2 \\ \text{subject to} & 5x_1' + 8x_2 + x_3 = 130 \\ & -x_1' + 2x_1 - x_4 = -6 \\ & x_1', x_2, x_3, x_4 \geq 0. \end{array}$$

The other way is to first replace x_1 by $x_1' - x_1''$, with $x_1', x_1'' \geq 0$. Then we introduce a new nonnegative variable x_5, such that $x_1' - x_1'' + x_5 = 10$. This also works but uses more variables than the previous one.

(f) We first change the objective function to minimizing $-x_1 - 2x_2 - 4x_3$. For the absolute value, remember than $|x| \leq y$ is equivalent to that both $x \leq y$ and $x \geq -y$ hold. So we can rewrite the LP as follows:

$$\begin{array}{ll} \text{Minimize} & -x_1 - 2x_2 - 4x_3 \\ \text{subject to} & 4x_1 + 3x_2 - 7x_3 \leq x_1 + x_2 + x_3 \\ & 4x_1 + 3x_2 - 7x_3 \geq -x_1 - x_2 - x_3 \\ & x_1, x_2, x_3, \geq 0. \end{array}$$

Simplifying the constraints we get

$$\begin{array}{ll} \text{Minimize} & -x_1 - 2x_2 - 4x_3 \\ \text{subject to} & 3x_1 + 2x_2 - 8x_3 \leq 0 \\ & 5x_1 + 4x_2 - 6x_3 \geq 0 \\ & x_1, x_2, x_3, \geq 0. \end{array}$$

Adding new variables x_4, x_5, we get the standard form:

$$\begin{array}{ll} \text{Minimize} & -x_1 - 2x_2 - 4x_3 \\ \text{subject to} & 3x_1 + 2x_2 - 8x_3 + x_4 = 0 \\ & 5x_1 + 4x_2 - 6x_3 - x_5 = 0 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0. \end{array}$$

3.9.6

(a) False. For example just take two disjoint unit disk, both of them are convex but their union is not convex, e.g. by taking the two centers x, y and then $(x + y)/2$ is their convex combination but does not belong to the union.

(b) False. For example we take the unit disk again. The complement is the whole plane minus the unit disk. Take a line segment intersecting the unit disk, with both ends outside. We can see this contradicts the convexity again.