Conference Program

Saturday (Nov 23) afternoon

1:00–2:00	Advances on Regularity Methods and Applications	Jacob Fox
2:00–3:00	Equiangular Lines with a Fixed Angle	Yufei Zhao
3:00–3:30	- - - -break- - - -	
3:30–4:00	Concentration Inequalities for Finding Rainbow Matchings	Andrey Kupavskii
4:00–5:00	Chromatic Numbers, Independent Sets, and Embedding Complexes	Florian Frick
5:00–6:00	Towards the Sunflower Conjecture	Shachar Lovett

Sunday (Nov 24) morning

9:00–10:00	Ramsey Numbers	Jacob Fox
10:00–10:30	New Results on Projections	Guy Moshkovitz
10:30–10:45	- - - -break- - - -	
10:45–11:45	Independent Sets in the Hypercube Revisited	Will Perkins
11:45–12:15	Rainbow Matchings for 3-uniform Hypergraphs	Xiaofan Yuan

Coffee, refreshments and snacks will be provided during the break. A light breakfast will be served on Sunday morning starting from 8:30am.

Abstracts

Advances on Regularity Methods and Applications

Jacob Fox (Stanford)

Szemerédi's regularity lemma and its variants are some of the most powerful tools in combinatorics. For example, Szemerédi used an early version in the proof of his celebrated theorem on long arithmetic progressions in dense sets of integers. It has also played a central role in extremal combinatorics, additive combinatorics, property testing, and graph limits. The regularity lemma roughly says that the vertex set of every graph can be partitioned into a bounded number of parts such that for most of the pairs of parts, the bipartite graph between the pair is quasirandom. A substantial drawback in using the regularity lemma is that it typically gives enormous, tower-type bounds in its various applications. A major program over the last few decades has been to find alternative proofs of the many applications of the regularity lemma with much better quantitative bounds. We will discuss recent advances on the program, including some big successes of the program, as well as the first examples of applications in which the tower-type bounds which come from applying a regularity lemma is necessary.

Equiangular Lines with a Fixed Angle

Yufei Zhao (MIT)

Solving a longstanding problem on equiangular lines, we determine, for each given fixed angle and in all sufficiently large dimensions, the maximum number of lines pairwise separated by the given angle.

A key ingredient is a new result in spectral graph theory: the adjacency matrix of a connected bounded degree graph has sublinear second eigenvalue multiplicity.

Joint work with Zilin Jiang, Jonathan Tidor, Yuan Yao, and Shengtong Zhang.

Concentration Inequalities for Finding Rainbow Matchings

Andrey Kupavskii (Institute for Advanced Study)

Consider a k-partite k-uniform hypergraph on $[n]^k$. It is not difficult to see that any such hypergraph with more than $(s-1)n^{k-1}$ edges contains a matching of size s. Aharoni and Berger asked a "transversal" variant of this question: given s hypergraphs, each having more than $(s-1)n^{k-1}$ edges, can we guarantee the existence of an s-matching with the i-th edge coming from the i-th hypergraph? In this talk, I will present our progress on this problem using a certain concentration inequality for the intersection of a family with a random matching. Joint work with Sergei Kiselev.

Chromatic Numbers, Independent sets, and Embedding complexes

Florian Frick (Carnegie Mellon University)

Topological methods have been successfully applied to combinatorial problems that benefit from detecting global obstructions. I will explain how chromatic numbers of graphs and questions about fair representations by independent sets can be seen as a more rigid variant of the topological problem of embedding simplicial complexes into Euclidean space. This topological viewpoint gives optimal results in the combinatorial setting in several interesting cases.

Towards the Sunflower Conjecture

Shachar Lovett (University of California, San Diego)

A sunflower with r petals is a collection of r sets so that the intersection of each pair is equal to the intersection of all. Erdos and Rado in 1960 proved the sunflower lemma: for any fixed r, any family of sets of size w, with at least about w^w sets, must contain a sunflower. The famous sunflower conjecture is that the bound on the number of sets can be improved to c^w for some constant c. Despite much research, the best bounds until recently were all of the order of w^{cw} for some constant c. In this work, we improve the bounds to about $(\log w)^w$.

There are two main ideas that underlie our result. The first is a structure vs pseudo-randomness paradigm, a commonly used paradigm in combinatorics. This allows us to either exploit structure in the given family of sets, or otherwise to assume that it is pseudo-random in a certain way. The second is a duality between families of sets and DNFs (Disjunctive Normal Forms). DNFs are widely studied in theoretical computer science. One of the central results about them is the switching lemma, which shows that DNFs simplify under random restriction. We show that when restricted to pseudo-random DNFs, much milder random restrictions are sufficient to simplify their structure.

Joint work with Ryan Alweiss, Kewen Wu and Jiapeng Zhang.

Ramsey Numbers

Jacob Fox (Stanford University)

Given graphs H and G, the Ramsey number $r(H, G)$ is the minimum N such that every red-blue coloring of the edges of the complete graph on N vertices contains a red copy of H or a blue copy of G. I will survey some recent results towards better understanding the growth of Ramsey numbers for graphs and hypergraphs.

New Results on Projections

Guy Moshkovitz (Institute for Advanced Study)

What is the largest number of projections onto k coordinates guaranteed in every family of m binary vectors of length n? This fundamental question is intimately connected with important topics and results in combinatorics and computer science (Turan numbers, the Sauer-Perles-Shelah Lemma, the Kahn-Kalai-Linial Theorem), and is generally wide open. We essentially settle the question for a wide range of parameters (linear k and sub-exponential m).

Based on joint work with Noga Alon and Noam Solomon.

Independent Sets in the Hypercube Revisited

Will Perkins (University of Illinois, Chicago)

We revisit Sapozhenko's classic proof on the asymptotics of the number of independent sets in the discrete hypercube and Galvin's follow-up work on weighted independent sets. We combine Sapozhenko's graph container methods with the cluster expansion and abstract polymer models, two tools from statistical physics, to obtain considerably sharper asymptotics and detailed probabilistic information about the typical structure of (weighted) independent sets in the hypercube. These results refine those of Korshunov and Sapozhenko and Galvin, and answer several questions of Galvin. Joint work with Matthew Jenssen.

Rainbow Matchings for 3-unifrom Hypergraphs

Xiaofan Yuan (Georgia Tech)

Kühn, Osthus and Treglown, and, independently, Khan proved that if H is a 3-uniform hypergraph with n vertices such that $n \in 3\mathbb{Z}$ and large, and the minimum vertex degree of H is greater than $\binom{n-1}{2} - \binom{2n/3}{2}$, then H contains a perfect matching. Huang, Loh, and Sudakov showed that if, for $1 \leq i \leq t$, where $t < n/(3k^2)$, $F_i \subseteq \binom{[n]}{k}$ and $|F_i| > \binom{n}{k} - \binom{n-t+1}{k}$, then $\{F_1, \ldots, F_t\}$ admits a rainbow matching. In this paper, we show that for $n \in 3\mathbb{Z}$ sufficiently large, if, for $i \in \{1, \ldots, n/3\}$, $F_i \subseteq \binom{[n]}{3}$ and $\delta_1(F_i) > \binom{n-1}{2} - \binom{2n/3}{2}$, then $\{F_1, \ldots, F_{n/3}\}$ admits a rainbow matching. This is joint work with Hongliang Lu and Xingxing Yu.