New Bounds for Point-curve Incidences in the Plane

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A classical problem in combinatorial geometry is to determine the maximum number of incidences between a set of m points and n curves in the plane. If the curves are lines, then Szemerdi and Trotter proved that there could be at most $O(m^{2/3}n^{2/3} + m + n)$ incidences, and this bound is tight. For other classes of curves, very few tight bounds are known. Work in this area progressed in the 80s and 90s, culminating in an incidence bound by Pach and Sharir in 1998 that applies to a very general class of curves. Since then, there have only been improvements for a few specific types of curves. In this talk I will discuss some new developments that improve upon Pach and Sharir's bound for a broad class of curves. A key innovation is the use of higher-dimensional incidence geometry, coupled with a new way of cutting collections of curves into segments so that the corresponding set of segments is better behaved than the original collection of curves. This is joint work with Micha Sharir.