

The Abelian Sandpile

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The Abelian Sandpile is a deterministic diffusion process for configurations of chips on the integer lattice. Given such a configuration, any vertex with at least 4 chips can *topple*, distributing one chip to each of its 4 lattice neighbors. The process stabilizes by toppling vertices until all sites have at most 3 chips.

This simple framework produces striking fractal structures which eluded explanation for many years. We will discuss a framework for understanding the behavior of the sandpile, which connects the chip-firing rule to Apollonian circle packings and certain discrete tilings of the plane.