STACKS HW7 - FINAL ASSIGNMENT

(1) Let C and D be categories and calculate very explicitly the 2-limit of the diagram

$$C \rightrightarrows D \tag{0.0.1}$$

(2) Show that a morphism $\mathcal{X} \to \mathcal{Y}$ of categories is a monomorphism (i.e., fully faithful) iff the diagonal is an equivalence.

(3) Let C be a site and let $X' \to X$ be a covering in C. Show that the category $Sh(X' \to X)$ is equivalent to the 2-limit of the diagram

$$\widetilde{X'} \rightrightarrows \widetilde{X''} \mathrel{\substack{\rightarrow\\\rightarrow\\\rightarrow}} \widetilde{X'''} \tag{0.0.2}$$

(4) Let $D \to C$ be a fibred category. Show that the maps $D(V) \to D(U)$ defined in class are functors, and that, for a pair of maps $U \to V \to W$, the composition of the functors $D(W) \to D(V) \to D(U)$ is isomorphic to $D(W) \to D(U)$.

(5) Prove the 2-Yoneda lemma.

(6) Let $\mathcal{X} \to \mathbf{Sch}$ be a fibered category. Show that if the fibers are setoids, then $\mathcal{X}$ is equivalent to $\mathbf{Sch}_{/F}$ for some functor F. Show that in this case F is a sheaf iff $\mathcal{X} \to \mathbf{Sch}$ is a stack.