

## STACKS HW5 - COMMA CATEGORY AND ADJUNCTION

- (1) Let  $C$  be a site, and let  $X$  be an object in  $C$ . Recall that the comma category  $C/X$  inherits the structure of a site. Assume that  $C$  is subcanonical (which means that for every  $X \in C$ ,  $h_X$  is a sheaf).
- (a) Show that there is an equivalence of categories between  $Sh(C/X)$  and  $Sh(C)/h_X$ .
  - (b) Show that  $j^*: Sh(C) \rightarrow Sh(C)/h_X$ , given by  $F \mapsto (F \times h_X \xrightarrow{p_2} h_X)$  commutes with finite limits and has a right adjoint  $j_*$ . (Describe  $j_*$  explicitly.)