# STACKS HW4 - SHEAVES AND TOPOI

(1) Let $T$ be a topos. Define a topology $\tau$ on $T$ by declaring a morphism $F' \to F$ to be a covering if it is a surjection of sheaves. (This is called the canonical topology.)
   (a) Show that the canonical topology is a topology.
   (b) Show that the associated topos is equivalent to $T$.
   (c) Show that the canonical topology is the largest topology preserving the topos; i.e., prove that if $p\colon X' \to X$ is a morhpism and if every sheaf $F$ in $T$ satisfies the sheaf axiom with respect to $p$, then $p$ is an epimorphism in $T$.

(2) Define a site $C$ to be *subcanonical* if for every object $X \in C$, $h_X$ is a sheaf. (So, for instance, the etale site of a scheme is subcanonical.) Give an example of a site $C$ which is not subcanonical.

(3) Let $X' \to X$ be an étale surjection of affine schemes and let $Y$ be a scheme. Show that the 'first sheaf axiom' is satisfied, i.e., that the map

$$h_Y(X) \to h_Y(X')$$

is injective. (Assume that we already know this for $Y$ affine.)

(4) Let $F\colon C \to D$ and $G\colon D \to C$ be a pair of functors. We say that $F$ is *left adjoint* to $G$ if there is an isomorphism of (bi)-functors

$$\mathrm{Hom}(F(-), -) \cong \mathrm{Hom}(-, G(-)).$$

Let $C'$ be a category and let $C$ be a subcategory. Denote by $i$ the inclusion $C \to C'$, and suppose that $i$ has a left adjoint $a\colon C' \to C$. (For instance, $a$ could be sheafification.) Let $D\colon I \to C$ be a diagram.

Prove that $a(\varinjlim i \circ D) = \varinjlim D$