STACKS HW3 - ÉTALE MORPHISMS AND SHEAVES

(1) Think of lots of examples and non examples of étale morphisms, and work out the details as explicitly as you can.
(2) Show that the sheaf axiom of Hartshorne is equivalent to our sheaf axiom.
(3) Let C be a category and let $D\colon I \to \mathrm{Fun}(C^{op}, \mathbf{Sets})$ be a diagram.
  (a) Show that $\varprojlim D$ is the functor $X \mapsto \varprojlim D(i)(X)$.
  (b) Show that $\varinjlim D$ is the functor $X \mapsto \varinjlim D(i)(X)$.
(4) Let C be a category and let $D\colon I \to \widetilde{\mathbf{Sch}}$ be a diagram, where $\widetilde{\mathbf{Sch}}$ is the subcategory of sheaves in $\mathrm{Fun}(C^{op}, \mathbf{Sets})$.
  (a) Show that $\varprojlim D$ (in the category of sheaves) is the functor $X \mapsto \varprojlim D(i)(X)$.
  (b) Show that $\varinjlim D$ (in the category of sheaves) is the *sheafication* of the functor $X \mapsto \varinjlim D(i)(X)$.
(5) Let F be a sheaf, F' be a presheaf, and $f\colon F' \to F$ be an injection (as presheaves). Show that the sheafication of F' is isomorphic to F if and only if every section of F is locally in the image of f.