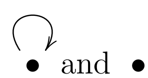
# STACKS HW1 - FUNCTORS

(1) Prove a slightly more general version of Yoneda's Lemma – let $C$ be a category, $X \in C$ an object, $h_X$ the functor $h_X(T) = Hom_C(T, X)$, and $F\colon C^{op} \to \textbf{Sets}$ a functor. Then $Hom(h_X, F) \cong F(X)$.

(2) Play the game "find the representing object" whenever you get the chance. Determine if the following functors are representable. If they are, find the representing object.
  (a) The functor $\text{Top}^{\text{op}} \to \textbf{Sets}$ taking a topological space $X$ to the set of open subsets of $X$.
  (b) The functor $\text{Top}^{\text{op}} \to \textbf{Sets}$ taking a topological space $X$ to the set of closed subsets of $X$.
  (c) The functor $\text{Top}^{\text{op}} \to \textbf{Sets}$ taking a topological space $X$ to the open subsets of $X$ whose complement is also open.
  (d) The functor $\textbf{HausTop}^{\text{op}} \to \textbf{Sets}$ taking a topological space $X$ to the set of open subsets of $X$. (**HausTop** is the category of Hausdorff topological spaces.)
  (e) The functor $\mathbb{A}^n - \{(0, \ldots, 0)\}\colon (\textbf{Sch}^{\text{op}}) \to (\textbf{Sets})$ taking a scheme $T$ to $\{(f_1, \ldots, f_n) \in \mathcal{O}_T(T)^n | \text{the } f_i \text{ do not all simultaneously vanish at the origin}\}$.
  (f) The functor $(\mathbb{A}^n - \{(0, \ldots, 0)\})/\mathbb{G}_m\colon \textbf{Sch}^{\text{op}} \to \textbf{Sets}$ taking a scheme $T$ to $(\mathbb{A}^n - \{(0, \ldots, 0)\})(T)/\sim$, where $\sim$ is the equivalence relation $(f_1, \ldots, f_n) \sim (f_1', \ldots, f_n')$ if there is a unit $u \in \mathcal{O}_T(T)$ such that $f_i' = uf_i$ for each $i$.

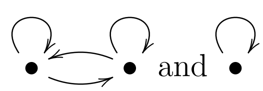
(3) Give 3 examples of equivalences of categories that are not isomorphisms of categories.

(4) Are the categories




equivalent?

(5) Are the categories




equivalent?