Jan 11 Math 250

DZB and class

January 2022

Definition. We say that an integer n is even if there exists an integer m such that $n = 2m$.

Examples. Is 6 even? Let's check the definition. Is there an integer m such that $6 = 2m$? Sure. If $m = 3$, then $6 = 2m$. So 6 is even.

What about $n = 7$? Well, if $7 = 2m$, then $m = 7/2 = 3.5$, which is not an integer.

What about $n = -8$? Sure. $-8 = 2(-4)$. Since -4 is an integer, -8 is even.

What about 0? Is 0 even? Can we solve the equation $0 = 2m$ for m? Yes: $m = 0$ is a solution. So 0 is even.

(A "lemma" is usually a "small" piece of math. A theorem is a big piece. This is subjective.)

Lemma. Let a and b be integers. Suppose that a and b are both even. Then $a + b$ is also even.

Proof. Since a is even, there exists an integer m such that $a = 2m$. Since b is even, there exists an integer n such that $b = 2n$. Then, $a + b = 2m + 2n$. By the distributive property of multiplication and addition, $a + b = 2(m + n)$. Since $m + n$ is an integer, we conclude that $a + b$ is even.

Note. The reason we can conclude that $a + b$ is even is that we verified the definition of even on the previous line.

Put math between dollar signs.

a + b vs $a + b$

This appears inline $\int_0^\infty e^{-x^2} dx$. To 'display' the equation, use double dollar signs

$$\int_0^\infty e^{-x^2} dx$$

$\alpha\beta\gamma\Gamma\sum\times\oplus\otimes\bigoplus$

The structure of proofs. The 'proof technique' of the "even + even = even" proof is a 'direct proof'.

There are four steps.

1. Find your hypotheses and assume them.
2. Write out what your hypotheses mean; i.e., write out the definitions of everything. (I.e., unwind.)
3. Think of something; usually, manipulate the definitions.
4. Conclude. (Verify the definitions of the thing you want to prove.)

I.e., Assume, Unwind, Manipulate, Conclude (AUMC)

Definition. Let a and b be integers. We say that a is a multiple of b if there exists some integer c such that $a = bc$. In this case, we say that b divides a, and write $b \mid a$.

Lemma. Suppose that a and b are even integers. Then ab is a multiple of 4.

Proof. Since a is even, there exists an integer m such that $a = 2m$. Since b is even, there exists an integer n such that $b = 2n$. Then $ab = (2m)(2n) = 4(mn)$. (Since multiplication commutes.) Since mn is an integer, ab is a multiple of 4.