If you have **questions** you are welcome to unmute and interrupt, or ask in chat (publicly or privately is fine!).

If there are any **technical difficulties** (for example, I am writing offscreen) please let me know right away!

My **website** is http://www.math.emory.edu/~dzb/

The **course** website is https://www.math.emory.edu/~dzb/teaching/250Spring2022/

The **syllabus** is available here https://www.math.emory.edu/~dzb/teaching/250Spring2022/syllabus-math-250-spring-2022.pdf

This **Miro** board is available at https://miro.com/app/board/uXjVOXdsj8c=/

The O**verleaf** board is available at https://www.overleaf.com/read/rcpfxdbhdgsh

Office hours are Mondays, 4:30-5:30 via Zoom. **Link is on the syllabus**. If you cannot make it to these office hours, please email me to set up an alternative time. (Preferrably email me 24 hours in advance, and please suggest a big list of times that you are available.)

Rewrites

For all - - .

Some - - - $\forall \exists$

there exists at least one. $\exists$

Proof by Contradiction.

P

Prove that if $x+y>20$, then $x>10$ or $y>10$.

Proof. Proceed by contradiction. $\neg P$

Suppose $x+y>20$. Suppose $x\leq 10$ and $y\leq 10$. Then $x+y\leq 20$. This contradicts our assumption that $x+y>20$. Thus $x>10$ or $y>10$. ■

Q

(valid proof that $\neg P \Rightarrow Q$

We want to prove P.

"There are 2 <u>cases</u>": either

P is true or P is false.

(i.e. $\neg P$ is true)

If we can "eliminate" $\neg P$,

then the <u>only</u> possibility is that

P is true.

It is if $\neg P$ is false, then P is true.

Template for proof by contradiction

Want to prove P.

1. Assume $\neg P$.
2. Do stuff. (Give a proof that $\neg P \Rightarrow Q$)
3. Observe that Q is false. (or prove)
4. Conclude that P is true.

Prove that $x^2 - y^2 = 1$ has no positive integer solutions.

IS prove that (there do not exist positive integers x and y s.t. $x^2 - y^2 = 1$)

A: the negation is more useful as a hypothesis.

why use contradiction?

Proof: Proceed by contradiction. Assume that there do exist positive integers x and y s.t. $x^2 - y^2 = 1$. $\neg P$

Then $(x-y)(x+y) = 1$. Then either

$x - y = 1$ and $x + y = 1$, or

$x - y = -1$ and $x + y = -1$.

In the 1^{st} case, adding gives $2x=2$, thus $x=1$. Then $-y=0$, so $y=0$. This contradicts that y is positive. In the 2^{nd} case, adding gives $2x=-2$, thus $x=-1$. This contradicts positivity of x. We conclude that there are no such x and y. $\square$

2 of 3

We proved $\neg P \implies (y=0 \text{ or } x=-1)$

$$\neg P \implies (Q_1 \text{ or } Q_2)$$

Both Q_1 and Q_2 are false.

Hieroglyph:

$$\left(\left(\neg P \Rightarrow Q \right) \wedge \neg Q \right) \Rightarrow P$$

Step 1 + 2

Step 3

Note: You can parse this) w/ a truth table

Prove that the equation $x^2 = 4y+3$ has no integer solutions.

<u>Proof:</u> Proceed by contradiction.

Assume that there exist integers x and y s.t. $x^2 \overset{(*)}{=} 4y+3$.

$\neg P$

Either x is even or x is odd.

If x is even, then x^2 is even.

(Then the LHS of $(*)$ is even,
but the RHS is odd). Thus
is a contradiction. Q.

If x is odd, then there is an integer t s.t. $x=2t+1$.

Then $(2t+1)^2 = 4t^2+4t+1 = 4y+3$,

then $(4t^2+4t-4y=2.)^{Q2}$ This is a contradiction because the LHS is divisible by 4 and the RHS is not. $\blacksquare$

This is a valid proof that

$\neg P \implies (Q_1 \vee Q_2)$.

Bof Q_1 and Q_2 are both false.

Thus $\neg P$ is false,

Thus P is true.

Euclid's theorem: there are infinitely many prime numbers.

Note: $7+1=8$ not prime

$11+1=12$

Proof: Proceed by contradiction.

Assume that there are only finitely many primes.

Label them as $p_1, p_2, \dots, p_r$ the final prime.

with $p_1 < p_2 < \dots < p_r$.

(IE $p_1 = 2$ $p_2 = 3$ $p_3 = 5$ $p_4 = 7, \dots$)

Then let $N = p_1 p_2 \cdots p_r + 1$.

Note: N cannot be prime, because $N > p_r$ and we assumed p_r was the final prime.

Since N is composite, it must have some prime factor

p_i. Since p_i appears in the product $p_1 \cdots p_r$, $p_i \mid p_1 \cdots p_r$ by transitivity. But $p_i \mid N$, so by the 2 out of 3 rule, $p_i \mid 1$. This is a contradiction, since 1 has no prime divisors. ∎

Warning: this doesn't prove

that $p_1 \cdots p_r + 1$ is prime.