If you have questions you are welcome to unmute and interrupt, or ask in chat (publicly or privately is fine!).

If there are any technical difficulties (for example, I am writing offscreen) please let me know right away!

My website is http://www.math.emory.edu/~dzb/

The course website is https://www.math.emory.edu/~dzb/teaching/250Spring2022/

The syllabus is available here https://www.math.emory.edu/~dzb/teaching/250Spring2022/syllabus-math-250-spring-2022.pdf

This Miro board is available at https://miro.com/app/board/uXjVOXdsj9M=/

The Overleaf board is available at https://www.overleaf.com/read/ymgbgzsfcrrd

Office hours are Mondays, 4:30-5:30 via Zoom. Link is on the syllabus. If you cannot make it to these office hours, please email me to set up an alternative time. (Preferrably email me 24 hours in advance, and please suggest a big list of times that you are available.)

Modifying statements + Boolean Algebra

Use variables to denote and talk about statements.

Usually: P, Q, R, S

	P	Q	
And $P \wedge Q$			Both are true
OR $P \vee Q$			At least one true
Not $\neg P$			opposite of P.

Truth table

P	Q	$P \wedge Q$	$P \vee Q$	$\neg P$
T	T	T	T	F
F	T	F	T	T
T	F	F	T	
F	F	F	F	

Quantifiers: $P(x)$ is an open statement.

$\forall x, P(x)$ $\forall$ = for all

$\exists x$ such that $P(x)$ $\exists$ = there exists

"for some" or "for at least one"

$P(x) =$ " $x+1 > 0$ "

$\forall x$ (such that x is a real number), $x+1 > 0$.

$\boxed{\text{False}}$

$\exists x$ s.t. $x+1 > 0$. True

(such that)

\forall

\exists

Implications, or "If ... then ..."

If P, then Q

or

$P \Rightarrow Q$

P implies Q

Q if P

\Rightarrow

These all are different ways to say the same thing

$P \Rightarrow Q$

Replace $P \Rightarrow Q$ with

"Whenever P is true, Q is true"

Whenever $\boxed{x \text{ is real,}}$ $\boxed{x^2 \geq 0.}$

If a and b are even, then $a+b$ is even.

If P is false, ..., who cares.

P	Q	$P \Rightarrow Q$
T	T	T
F	T	T
T	F	F
F	F	T

To disprove $P \Rightarrow Q$, find an example where P is true and Q is false.

<u>Negations</u>:

P is true, $\neg P$ is false

The <u>negation</u> of P is the statement with the opposite truth value

Negation identities

$$\neg(P \vee Q) = \neg P \wedge \neg Q$$

We can prove identities using truth tables

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	T	F	$F \wedge F = F$
F	T	T	F	$T \wedge F = F$
T	F	T	F	$F \wedge T = F$
F	F	F	T	$T \wedge T = T$

P	Q	R	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T
T	T	F	T	T ∧ T = T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$$\forall x, P(x) = \exists x \text{ s.t. } \neg P(x)$$

$$\neg(\forall x, x^2 > 0) = \exists x \text{ s.t. } x^2 \leq 0$$

$$\neg(\exists x \text{ s.t. } Q(x)) = \forall x, \neg Q(x)$$

Think about opposites.

$$\boxed{\neg(P \Rightarrow Q) \equiv P \wedge \neg Q}$$

<u>Note:</u> the negation of an implication is <u>not</u> an implication.

$$\neg(P \Rightarrow Q) \neq \neg P \Rightarrow \neg Q$$

$$\neq Q \Rightarrow P \quad \text{(converse)}$$

$$\neg\left(\boxed{P} \Rightarrow \boxed{(Q \wedge R)}\right)$$

$$P \wedge \neg(Q \wedge R) =$$

$$P \wedge (\neg Q \vee \neg R) =$$

$$(P \wedge \neg Q) \vee (P \wedge \neg R)$$

$$\neg(\forall x, P(x) \vee Q(x))$$

$$\exists x \text{ s.t. } \neg(P(x) \vee Q(x)) =$$

$$\exists x \text{ s.t. } \neg P(x) \wedge \neg Q(x)$$

$P \Rightarrow Q$

$Q \Rightarrow P$ converse

$P \Leftrightarrow Q$ both are true

if a and b are even, then a+b is even

a and b even $\Rightarrow$ a+b even T

conv a+b even $\Rightarrow$ a and b even.

F b/c if a=b=1, a+b is even, a+b are not.

Contrapositive of $P \Rightarrow Q$ is

$\neg Q \Rightarrow \neg P$.

They both have the same negation,

$$\neg(\boxed{\neg Q} \Rightarrow \boxed{\neg P}) =$$

$$\neg Q \wedge \neg(\neg P) = \neg Q \wedge P$$

$$= P \wedge \neg Q = \neg(P \Rightarrow Q)$$