

**MATH 250 HANDOUT 12 - COMPOSITIONS AND
INJECTIVITY/SURJECTIVITY**

- (1) Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be the function $f(x) = \frac{1}{1+x^2}$ and let $g: \mathbf{R} \rightarrow \mathbf{R}$ be the function $g(x) = e^x$.
- (a) What is $g \circ f(0)$?
 - (b) What is $f \circ g(0)$?
 - (c) Give a formula for $f \circ g$ and $g \circ f$.
- (2) Let $f: \mathbf{R} \rightarrow \mathbf{Z}$ be the function $f(x) = \lfloor x \rfloor$ (i.e., round x down to the nearest integer) and let $g: \mathbf{Z} \rightarrow \mathbf{Z}$ be the function $g(n) =$ ‘the number of distinct prime factors of n ’. (So $g(0) = g(1) = 0$, $g(4) = 1$, $g(6) = 2$)
- (a) What is $g \circ f(\pi)$?
 - (b) What is $g \circ f(91.1023124)$?
 - (c) Is $g \circ f$ injective? Surjective?
- (3) Let $f: \mathbf{Z} \rightarrow P(\mathbf{Z})$ be the function $f(n) = \{n\}$ and let $g: P(\mathbf{Z}) \rightarrow P(\mathbf{Z})$ be the function $g(S) = S \cap \{1\}$.
- (a) What is $g \circ f(0)$?
 - (b) What is $g \circ f(1)$?
 - (c) Give a formula for $g \circ f$.

- (4) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be functions. Prove or disprove each of the following:
- If f and g are injections, then gf is an injection.
 - If f and g are surjections, then gf is a surjection.
 - If f and g are bijections, then gf is a bijection.
 - If gf is an injection, then f and g are injections.
 - If gf is a surjection, then f and g are surjections.
 - If gf is a bijection, then f and g are bijections.
 - If gf is an injection, then f is an injection.
 - (HW) If gf is an injection, then g is an injection.
 - (HW) If gf is a surjection, then f is a surjection.
 - (HW) If gf is a surjection, then g is a surjection.
 - If gf is a bijection, then f is a bijection.
 - If gf is a bijection, then g is a bijection.
 - If gf is an injection and g is a bijection, then f is an injection.
- (5) Let $f: A \rightarrow B$ be a function. Let $X, Y \subset A$ and let $W, V \subset B$. Each of the following statements are false as stated. Which become true if we assume that f is injective or surjective? In each case (f is injective, or f is surjective), prove your assertion or give a counterexample.
- $X \subset Y \Leftrightarrow f(X) \subset f(Y)$.
 - (HW) $f(X \cap Y) \supset f(X) \cap f(Y)$.
 - $f(X) - f(Y) \supset f(X - Y)$.
 - $X \supset f^{-1}(f(X))$.
 - $W \subset f(f^{-1}(W))$.
 - $V \subset W \Leftrightarrow f^{-1}(V) \subset f^{-1}(W)$.