

## MATH 250 HANDOUT 11 - INJECTIVITY

(1) Draw a picture of a function which is  
(a) injective

(b) not injective

(2) Finish the following sentence: a function  $f: X \rightarrow Y$  is *not* injective if ...

(3) Which of the following functions are injective?

(a) Inj  $\mathbf{N}$ :  $\mathbf{R} \rightarrow \mathbf{R}; x \mapsto x^2$ .

(b) Inj  $\mathbf{N}$ :  $\mathbf{R} \rightarrow \mathbf{R}; x \mapsto \frac{x+1}{2}$ .

(c) Inj  $\mathbf{N}$ :  $\mathbf{R} \rightarrow \mathbf{R}; x \mapsto \cos x$ .

(d) Inj  $\mathbf{N}$ :  $[0, \pi) \rightarrow \mathbf{R}; x \mapsto \cos x$ .

(e) Inj  $\mathbf{N}$ :  $\mathbf{R}_{\geq 0} \rightarrow \mathbf{R}; x \mapsto \frac{x-1}{x+1}$ .

(f) Inj  $\mathbf{N}$ :  $\mathbf{R} - \{-1\} \rightarrow \mathbf{R}; x \mapsto \frac{x-1}{x+1}$ .

(g) Inj  $\mathbf{N}$ :  $\mathbf{R} \rightarrow \mathbf{R}; x \mapsto \arctan x$ .

(h) Inj  $\mathbf{N}$ :  $\mathbf{R}^3 \rightarrow \mathbf{R}^2; (x, y, z) \mapsto (x, y)$ .

(i) Inj  $\mathbf{N}$ :  $\mathbf{R}^2 \rightarrow \mathbf{R}^3; (x, y) \mapsto (x + y, x - y, x^2 + y^2)$ .

(j) Inj  $\mathbf{N}$ :  $P(\mathbf{R}) \rightarrow P(\mathbf{Z}); S \mapsto S \cap \mathbf{Z}$ .

(k) Inj  $\mathbf{N}$ :  $P(\mathbf{Z}) \rightarrow P(\mathbf{Z}); S \mapsto S \cup \{1\}$ .

(l) Inj  $\mathbf{N}$ :  $\mathbf{Z} \rightarrow P(\mathbf{Z}); n \mapsto \{n\}$ .

(m) Inj  $\mathbf{N}$ :  $P(\mathbf{Z}) \rightarrow \mathbf{Z}; S \mapsto |S|$  if  $S$  is finite, 0 if  $S$  is infinite.

(n) Inj  $\mathbf{N}$ :  $\mathbf{R} \rightarrow \mathbf{R}; x \mapsto x^3 + 1$ .

(o) Inj  $\mathbf{N}$ :  $\mathbf{R} \rightarrow \mathbf{R}; x \mapsto x(x^2 - 1)$ .

(p) Inj  $\mathbf{N}$ :  $\mathbf{C} \rightarrow \mathbf{C}; x \mapsto x^2$ .

(q) Inj  $\mathbf{N}$ :  $[1, \infty) \rightarrow [0, \infty); x \mapsto x^3 - x$ .