MATH 250 HANDOUT 7 - MORE PROBLEMS ABOUT SETS; POWER SETS

(1) Let A, B, and C be sets. Draw a Venn diagram demonstrating each of the following and then prove each of the following.
   (a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
   (b) Prove that $A - (B \cap C) = (A - B) \cup (A - C)$.
   (c) Prove that $(A - B) \cup (B - A) \subset (A \cup B) - (A \cap B)$.
   (d) Prove that if $A - B \subset C$, then $\overline{C} \subset \overline{A} \cup B$.
   (e) Prove that if $A \subset B$, then $A \cup B - A \cap B = A - B$.

(2) Disprove each of the following statements by giving examples of sets A, B, C demonstrating that the statement is false. (I.e., give a counterexample.)
   (a) For all sets A, B, C, $A \cup (B \cap C) = (A \cup B) \cap C$.
   (b) For all sets A, B, C, $A \cap (B \cup C) = (A \cap B) \cup C$.
   (c) For all sets A, B, C, if A, B are subsets of C, then $(C - A) - B = C - (A - B)$.
   (d) For all sets A, B, C, if A, B are subsets of C and if $A \subset B$, then $(C - A) \subset (C - B)$.

Power Sets

(a) Write out 4 elements of
  (i) $P(\{0, 1, 2, 3, 4\})$;
  (ii) $P(\mathbf{N})$;
  (iii) $P(\mathbf{R})$.

(b) Let $A = \{0, 1, 2\}$. Circle whichever of the following statements are true.
  (i) $\{0\} \subset P(A)$;
  (ii) $\{1, 2\} \in P(A)$;
  (iii) $\{\{0, 1\}, \{1\}\} \subset P(A)$;
  (iv) $\emptyset \in P(A)$;
  (v) $\emptyset \subset P(A)$;
  (vi) $\{\emptyset\} \in P(A)$.
  (vii) $\{\emptyset\} \subset P(A)$;
  (viii) $\{1, \{1\}\} \subset P(A)$.

(c) Let A and B be sets. Prove or disprove:
  (i) $P(A) \cup P(B) \subset P(A \cup B)$.
  (ii) $P(A \cup B) \subset P(A) \cup P(B)$.
  (iii) $P(A) \times P(B) \subset P(A \times B)$.
  (iv) $A = B$ if and only if $P(A) = P(B)$.