MATH 250 HANDOUT 17 - BIJECTIONS

(1) For each of the following, give an example of a bijection. Once you have done this, prove that each of your bijections is in fact a bijection.
  (a) $\mathbb{Z} \to \mathbb{Z} \times \{1, 2\}$.
  (b) $2\mathbb{Z} \to 2\mathbb{Z} \times \{1, 2\}$.
  (c) $\mathbb{Z} \to \mathbb{Z} \times \{1, 2, 3\}$.
  (d) $\mathbb{O} \to \mathbb{E} \times \{1, 2, 3\}$.
  (e) $\mathbb{Z} \to d\mathbb{Z}$, where d is an integer.
  (f) $\mathbb{Z} \to \mathbb{Z}_{\geq 0}$.
  (g) $\mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$.
  (h) $\mathbb{Z} \to \mathbb{Q}$ (Hint: Modify the bijection $\mathbb{Z}_{>0} \to \mathbb{Q}_{>0}$ from class.)

(2) For each of the following, give an example of an injection or a surjection.
  (a) $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ (injection).
  (b) $\mathbb{Z}^d \to \mathbb{Z}$ (injection).
  (c) $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$ (surjection).

(3) Prove that the relation $A \sim B$ if there exists a bijection from A to B is an equivalence relation.